Unified formulation of single and two-moment normalizations of the rain drop size distribution based on the Gamma probability density function

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Basics

$$N(D) = \frac{N_t}{D_c} g(x) \quad with \quad x = \frac{D}{D_c}.$$

$$g(x;\lambda,\mu) = \frac{\lambda^{\mu+1}}{\Gamma(\mu+1)} x^{\mu} exp(-\lambda x).$$

$$N(D) = \frac{N_t}{D_c} g(x;\lambda,\mu) = \frac{N_t}{D_c}\left[\frac{\lambda^{\mu+1}}{\Gamma(\mu+1)}\left(\frac{D}{D_c}\right)^{\mu} exp\left(-\lambda\frac{D}{D_c}\right)\right].$$

$$D_c = D_{4,3} = \frac{M_4}{M_3}.$$

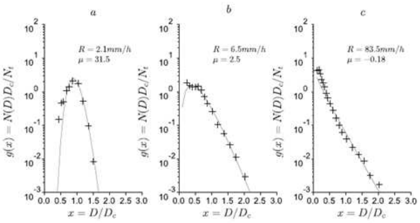
$$M_k = \int_0^{\infty} N(D) D^k dD = \frac{\Gamma(\mu+k+1)}{\Gamma(\mu+1)}\frac{N_t D_c^k}{\lambda^k}.$$

$$\lambda = \mu + 4,$$

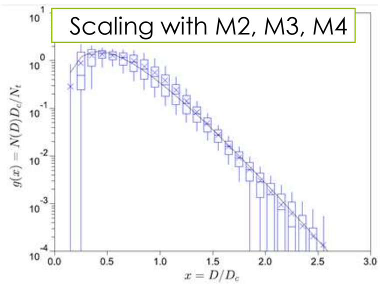
$$M_0^* = \frac{M_2\lambda^2}{(\mu+2)(\mu+1)D_c^2}$$

$$\mu = \frac{3-4A}{A-1} \quad with \quad A = \frac{M_3^2}{M_2 M_4}.$$

Fits for individual spectra ...



and for a 3-year DSD climatology



Comments

This study offers a unified formulation of single and double-moment normalizations of the raindrop size distribution (DSD), which have been proposed in the framework of the scaling analysis in the literature.

The key point is to consider a well-defined "general distribution" g(x) as the probability density function (pdf) of the raindrop diameter scaled by a characteristic diameter Dc. We use the ratio of the 4th to the 3rd DSD moments as the characteristic diameter and the two-parameter gamma pdf to model the g(x)-function.

The theory is illustrated with a three-year DSD time series collected with a Parsivel disdrometer, including a large variety of convective and non-convective rainfall events representative of the rainfall climatology in the Cevennes region, France. It is first shown that three DSD moments (M_2, M_3, M_4) allow to satisfactory model the DSD both for individual spectra and for time series of spectra.

The formulation is then extended to the one- and two-moment normalization by introducing single and dual power-law models between the explained moments (total concentration, characteristic diameter) and the scaling moment(s). Compared with previous scaling formulations, our approach explicitly accounts for the prefactors of the power-law models to yield a unique and dimensionless g(x), whatever the scaling moment(s) considered. A parameter estimation procedure, based on the analysis of the power-law regressions and the so-called self-consistency relationships, is proposed for both the one- and two-moment normalizations.

When implemented with contrasted scaling DSD moments (rainrate and/or radar reflectivity), the method yields g(x)-functions consistent with the one obtained with the three-moment normalization, although the 3-year DSD time series exhibits quite a large variability.

The intra-event variability of the DSD is illustrated for 22 October 2008 rain event: it is shown that very consistent g(x)-functions can be obtained for homogeneous rain phases, whatever the scaling moments used, and that the g(x)-functions may present contrasting shapes from one phase to another. This supports the idea that the g(x)-function is process-dependent and not unique as hypothesized in the scaling theory.

Single-moment normalization

$$N_t = C_i M_i^{\alpha_i},$$

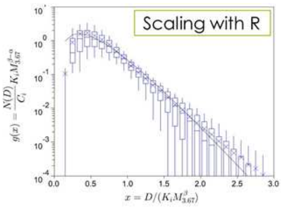
$$D_c = K_i M_i^{\beta_i}.$$

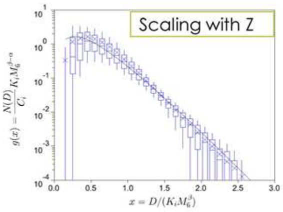
$$N(D) = \frac{C_i M_i^{\alpha_i}}{K_i M_i^{\beta_i}}\frac{\lambda^{\mu+1}}{\Gamma(\mu+1)}\left(\frac{D}{K_i M_i^{\beta_i}}\right)^{\mu} exp\left(-\lambda\frac{D}{K_i M_i^{\beta_i}}\right)$$

$$M_k = \frac{\Gamma(\mu+k+1)}{\Gamma(\mu+1)} C_i K_i^k \frac{M_i^{\alpha_i+k\beta_i}}{\lambda^k}$$

$$\alpha_i + i\beta_i = 1,$$

$$\frac{\Gamma(\mu+i+1)}{\Gamma(\mu+1)} C_i \left(\frac{K_i}{\lambda}\right)^i = 1.$$





Double-moment normalization

$$N_t = C_{ij} M_i^{\alpha_i} M_j^{\alpha_j},$$

$$D_c = K_{ij} M_i^{\beta_i} M_j^{\beta_j}.$$

$$N(D) = \frac{C_{ij} M_i^{\alpha_i} M_j^{\alpha_j}}{K_{ij} M_i^{\beta_i} M_j^{\beta_j}}\frac{\lambda^{\mu+1}}{\Gamma(\mu+1)}\left(\frac{D}{K_{ij} M_i^{\beta_i} M_j^{\beta_j}}\right)^{\mu} exp\left(-\lambda\frac{D}{K_{ij} M_i^{\beta_i} M_j^{\beta_j}}\right).$$

$$M_k = \frac{\Gamma(\mu+k+1)}{\Gamma(\mu+1)} C_{ij} K_{ij}^k \frac{M_i^{\alpha_i+k\beta_i} M_j^{\alpha_j+k\beta_j}}{\lambda^k}$$

$$\alpha_i + i\beta_i = 1,$$

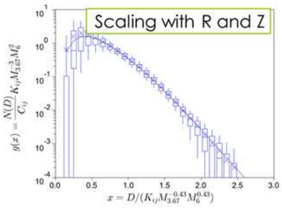
$$\alpha_j + i\beta_j = 0,$$

$$\alpha_i + j\beta_i = 0,$$

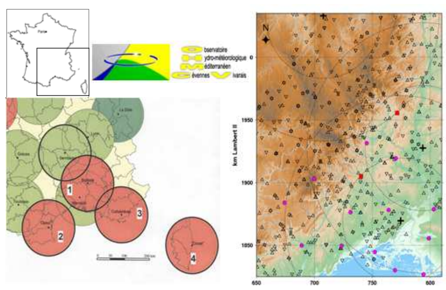
$$\alpha_j + j\beta_j = 1,$$

$$\frac{\Gamma(\mu+i+1)}{\Gamma(\mu+1)} C_{ij} \left(\frac{K_{ij}}{\lambda}\right)^i = 1,$$

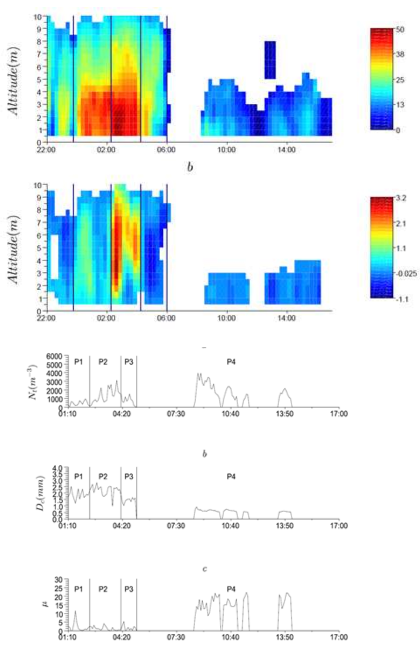
$$\frac{\Gamma(\mu+j+1)}{\Gamma(\mu+1)} C_{ij} \left(\frac{K_{ij}}{\lambda}\right)^j = 1.$$

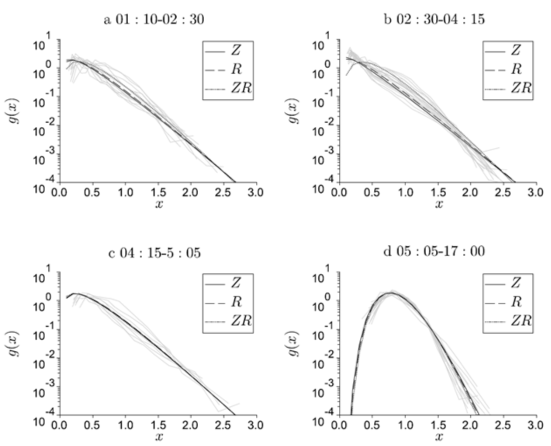


Normalization framework	μ
N_t and D_c	2.76
Z	1.47
R	2.00
ZR	2.80



DSD intra-event variability: the 22 October 2008 event





Normalization framework	phase 1	phase 2	phase 3	phase 4
Z	0.71	0.38	1.20	12.8
R	0.89	0.63	1.26	13.4
ZR	1.55	2.06	1.08	14.3

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