Bridging Ground Validation and Satellite Algorithms: DSD Working Group Progress Report
Using Scattering and Integral Tables to Incorporate Observed DSD Relationships into Satellite Algorithms

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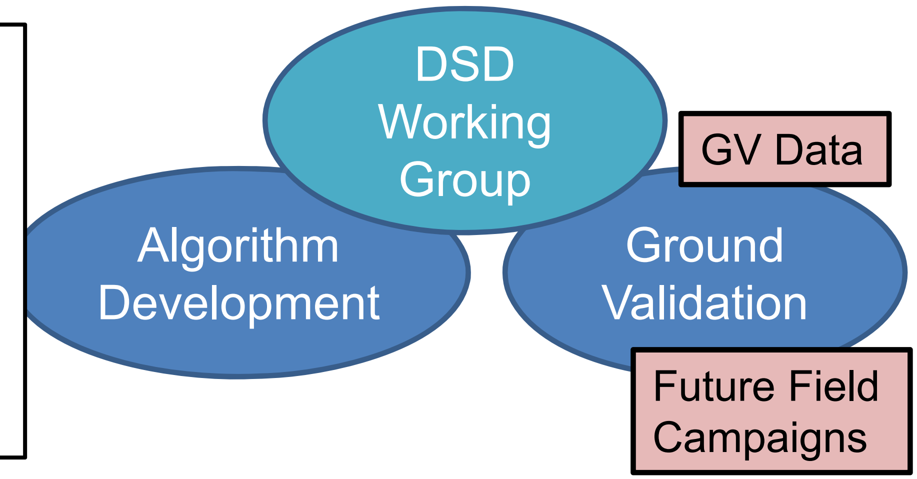
NASA GPM DSD Working Group: Bridging Algorithms and Ground Validation (GV)

General Objective: Use Ground Validation (GV) data to investigate relationships between DSD parameters that support, or guide, the assumptions used in satellite retrieval algorithms.
Rationale: Relationships between DSD parameters, if found, can be used to constrain the unknowns in satellite algorithms.



With guidance from Algorithm Developers, we are using previously collected GV data (point, columnar, and spatial GV data sets) to address these objectives:

1. Develop physically based relationships between DSD parameters.
2. Develop a framework to incorporate GV findings into Algorithms.

Discussed today

3. Describe the vertical structure of DSD parameters.
4. Investigate snow size parameters and their correlations.

Future Work

DSD Working Group Monthly Teleconference calls: 3rd Thursday @ 1 PM Eastern.

Define Gamma shaped DSD, N_w, D_m, μ:

$$N(D; N_w, D_m, \mu) = N_w f(\mu)\left(\frac{D}{D_m}\right)^{\mu} \exp\left(-\frac{(4+\mu)}{D_m} D\right)$$

Difficult to estimate μ and D_m from individual N(D) spectra because μ and D_m are correlated (Chandrasekar & Bringi 1989)

To avoid fitting artifacts, do not estimate gamma DSD parameters.
Find relationships between Mass Spectrum Parameters (no assumed DSD shape).

Mass Spectrum

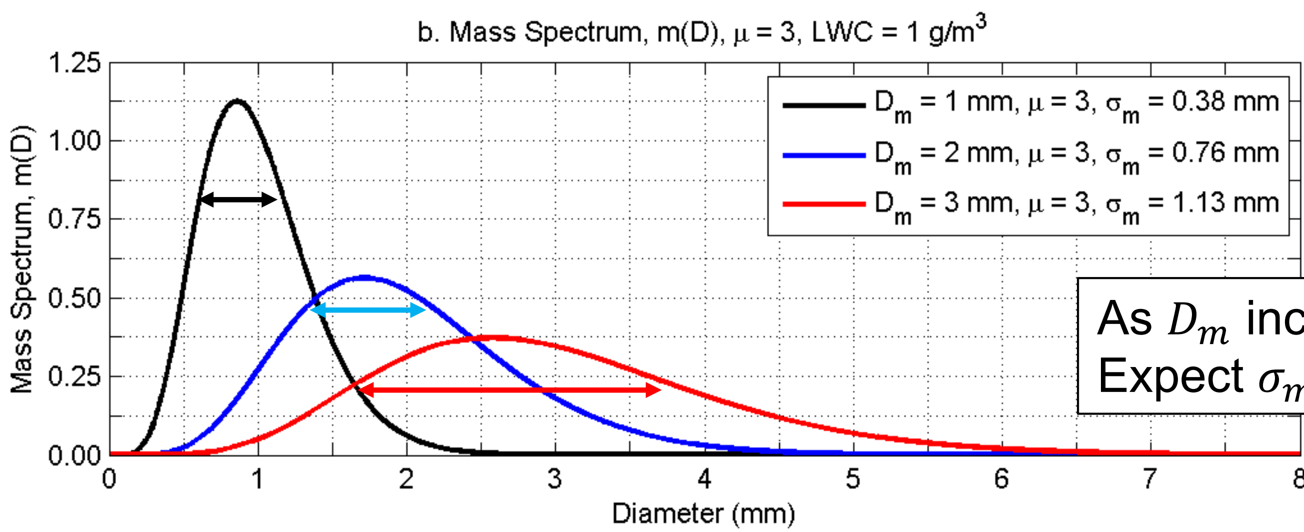
$$w(D) = \frac{\pi}{6 \cdot 10^3} \rho_w N(D) D^3$$

Mean Diameter

$$D_m = \frac{\sum_{D_{min}}^{D_{max}} w(D) D dD}{\sum_{D_{min}}^{D_{max}} w(D) dD}$$

Mass Spectrum Variance

$$\sigma_m^2 = \frac{\sum_{D_{min}}^{D_{max}} (D - D_m)^2 w(D) dD}{\sum_{D_{min}}^{D_{max}} w(D) dD}$$

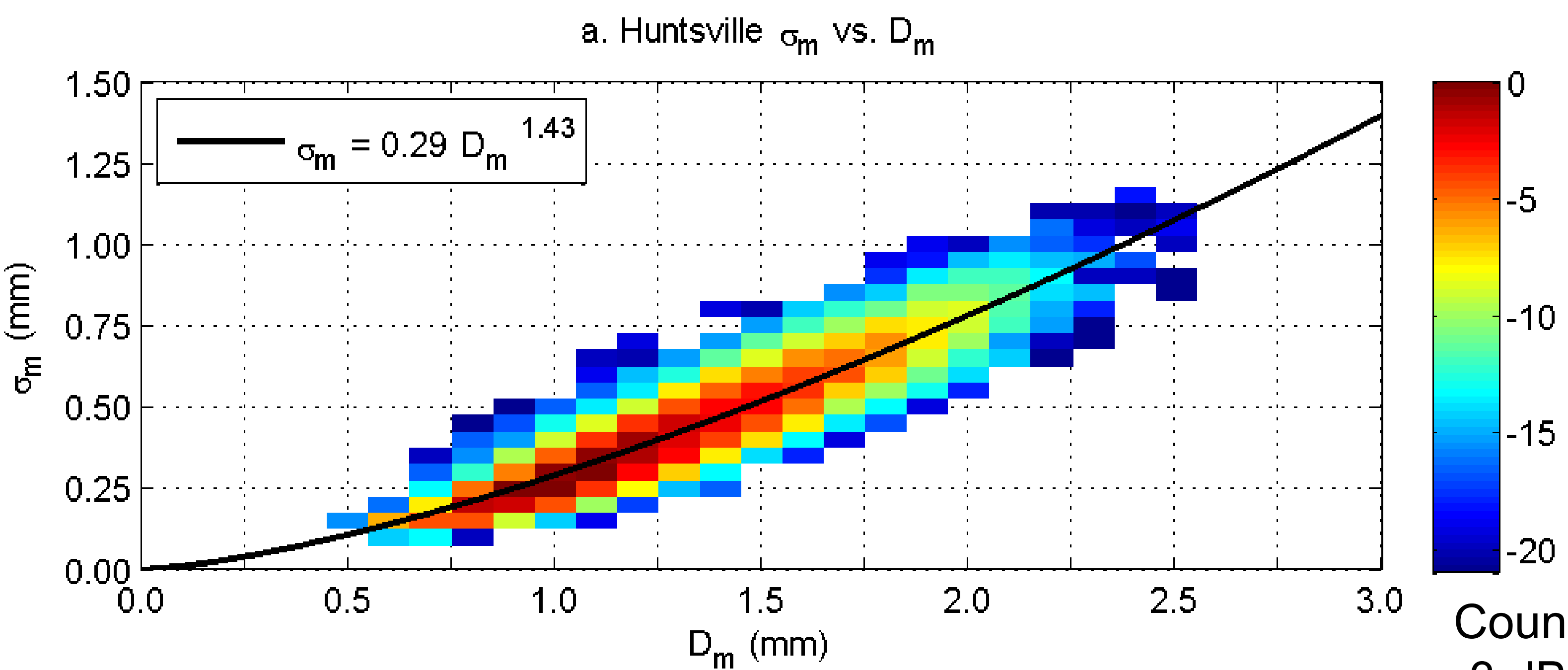


As D_m increases, Expect σ_m to increase

Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples

Frequency of Occurrence

- Observed σ_m & D_m
- No assumed DSD Shape
- Count is in dB
  - pixel with most counts = 0 dB
  - each -3 dB is half as many counts



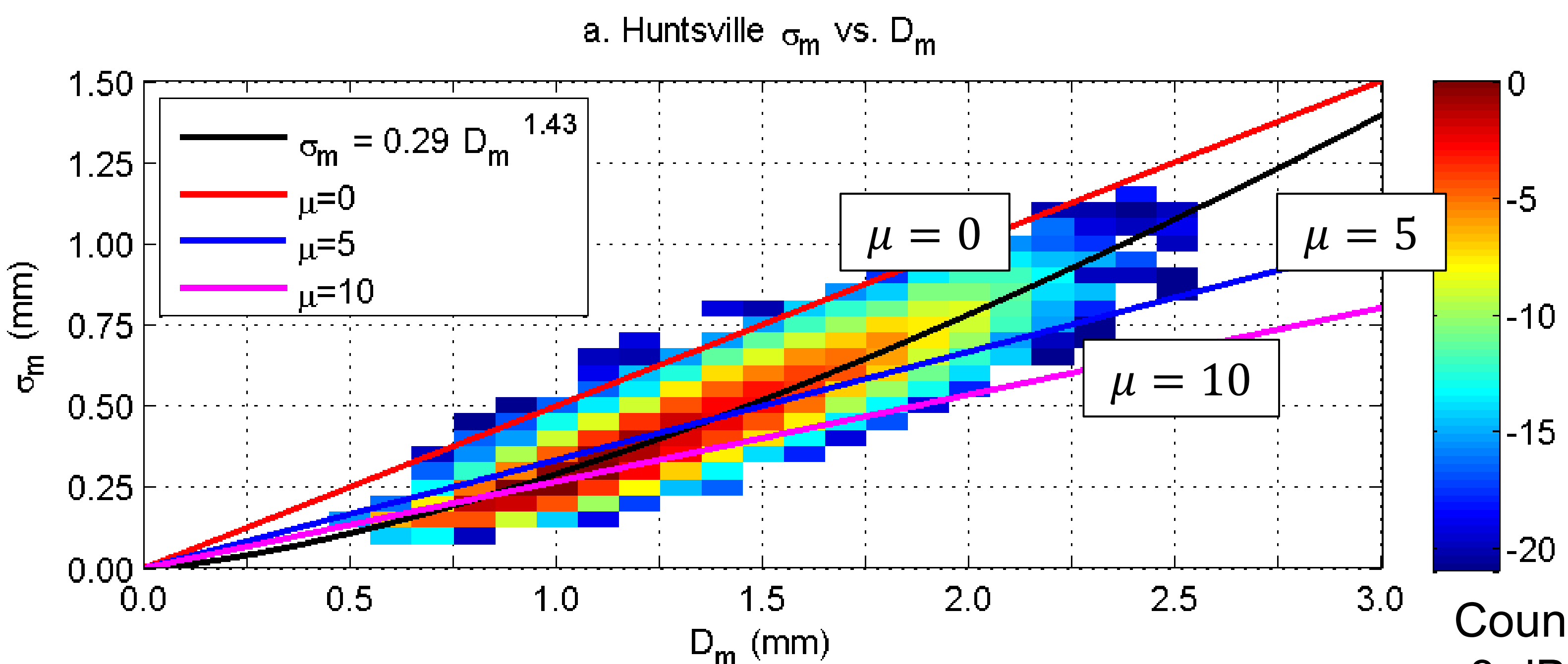
Count in dB
-3 dB = ½ cnt

Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples

If we assume a gamma shape DSD, there is a relationship between $\sigma_m - D_m - \mu$
(Assume the $D_{max} = \infty$)

1. Can estimate σ_m from D_m and μ $$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

2. Can estimate μ from D_m and σ_m $$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$



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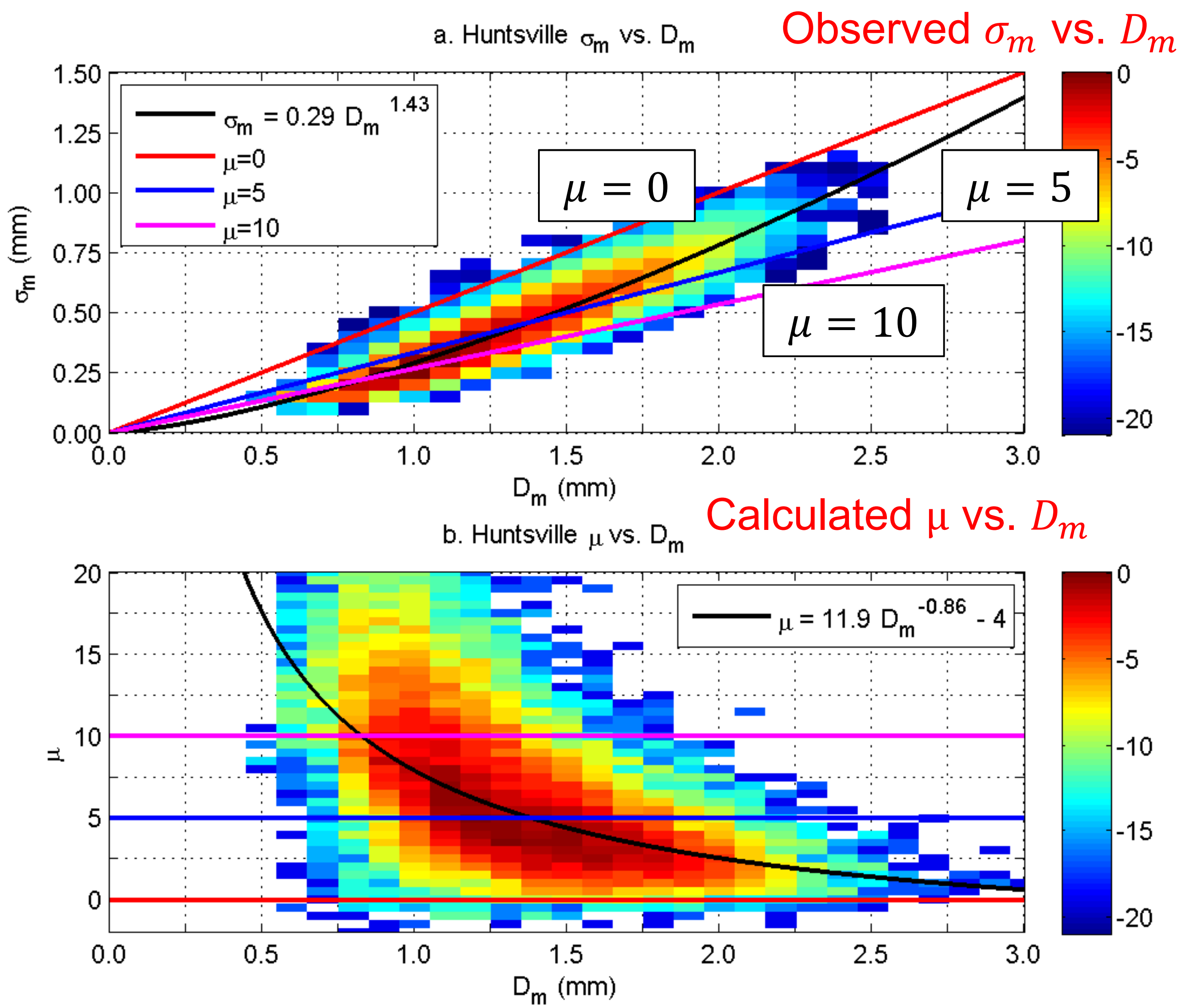


Estimate σ_m from D_m and μ using:

$$\sigma_m^2 = \frac{D_m^2}{\mu + 4}$$

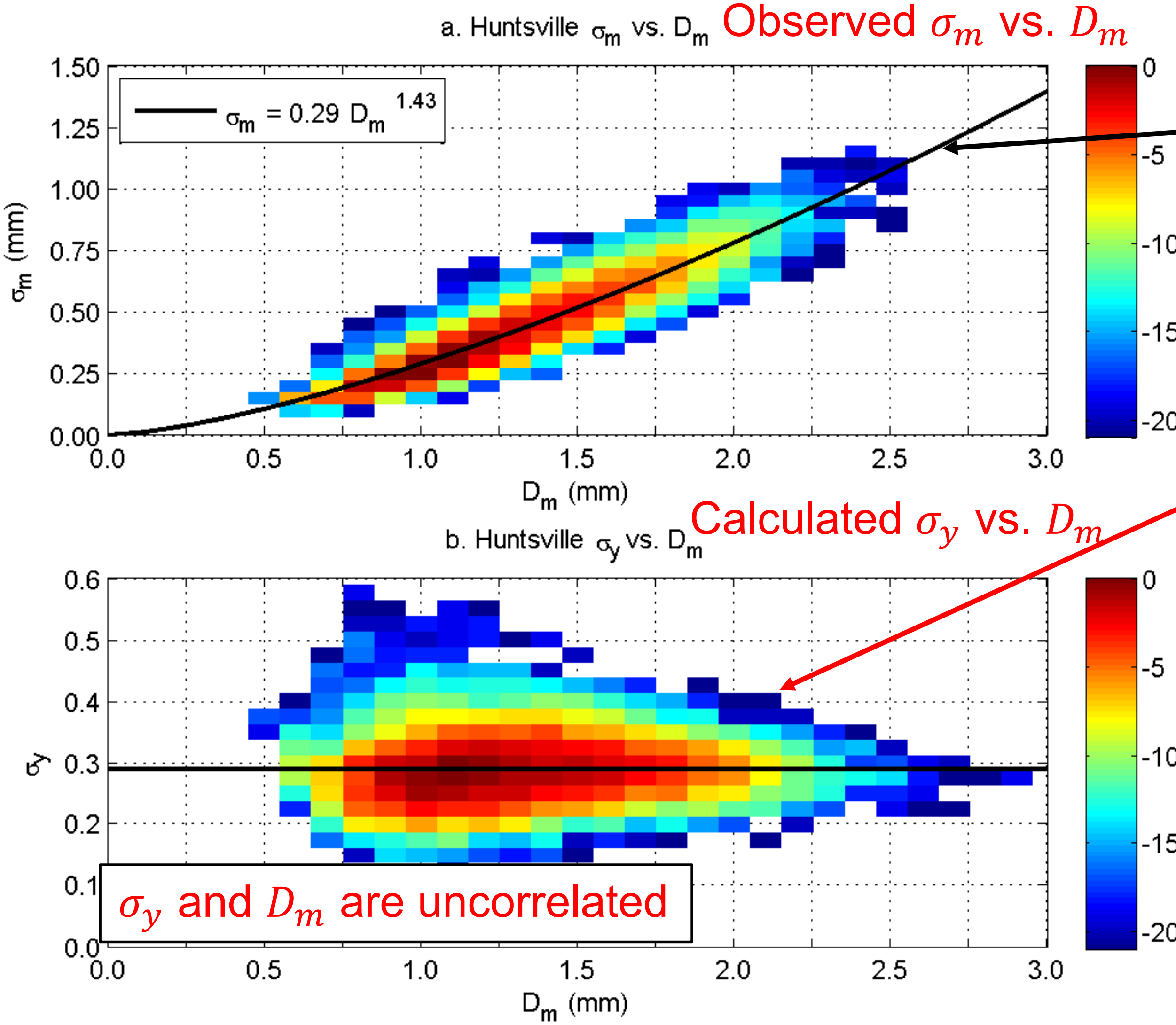
Estimate μ from D_m and σ_m using:

$$\mu = \frac{D_m^2}{\sigma_m^2} - 4$$

For this dataset, μ Power-law is:

$$\mu = \frac{11.9}{D_m^{-0.86}} - 4$$

Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples



How is the Power-law fit determined?

$\sigma_m = a\, D_m^b$

1. Define a new variable:

$$\sigma_y = \frac{\sigma_m}{D_m^b}$$ Observed

2. Adjust b, until σ_y & D_m are uncorrelated. For this data: $b = 1.43$

3. Coeff. $a = mean(\sigma_y)$:

$$a = \overline{\sigma_y} = 0.29$$

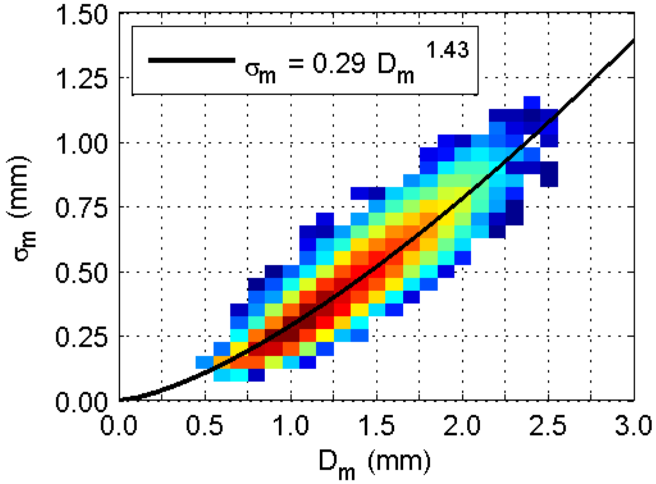
4. Power-law:

$$\sigma_m = \overline{\sigma_y} D_m^b = 0.29 D_m^{1.43}$$

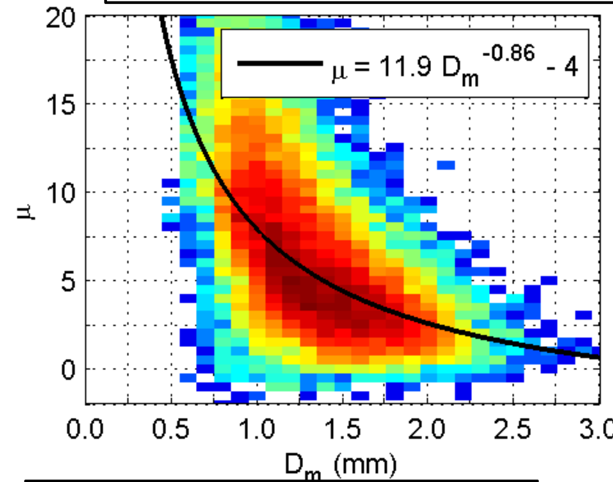
Haddad et al. (1996)

Huntsville, Alabama, three 2DVD disdrometers, 23 month deployment, 20,954 1-minute samples

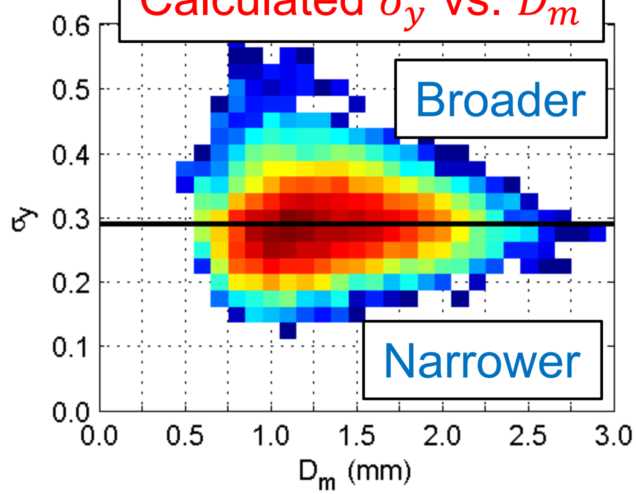
Observed σ_m vs. D_m



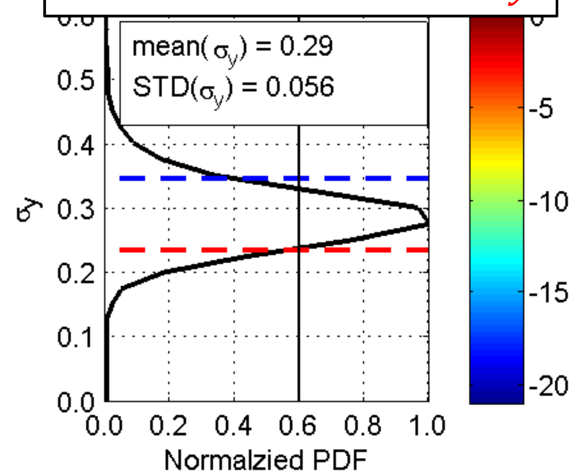
Calculated μ vs. D_m (assume a gamma DSD)



Calculated σ_y vs. D_m

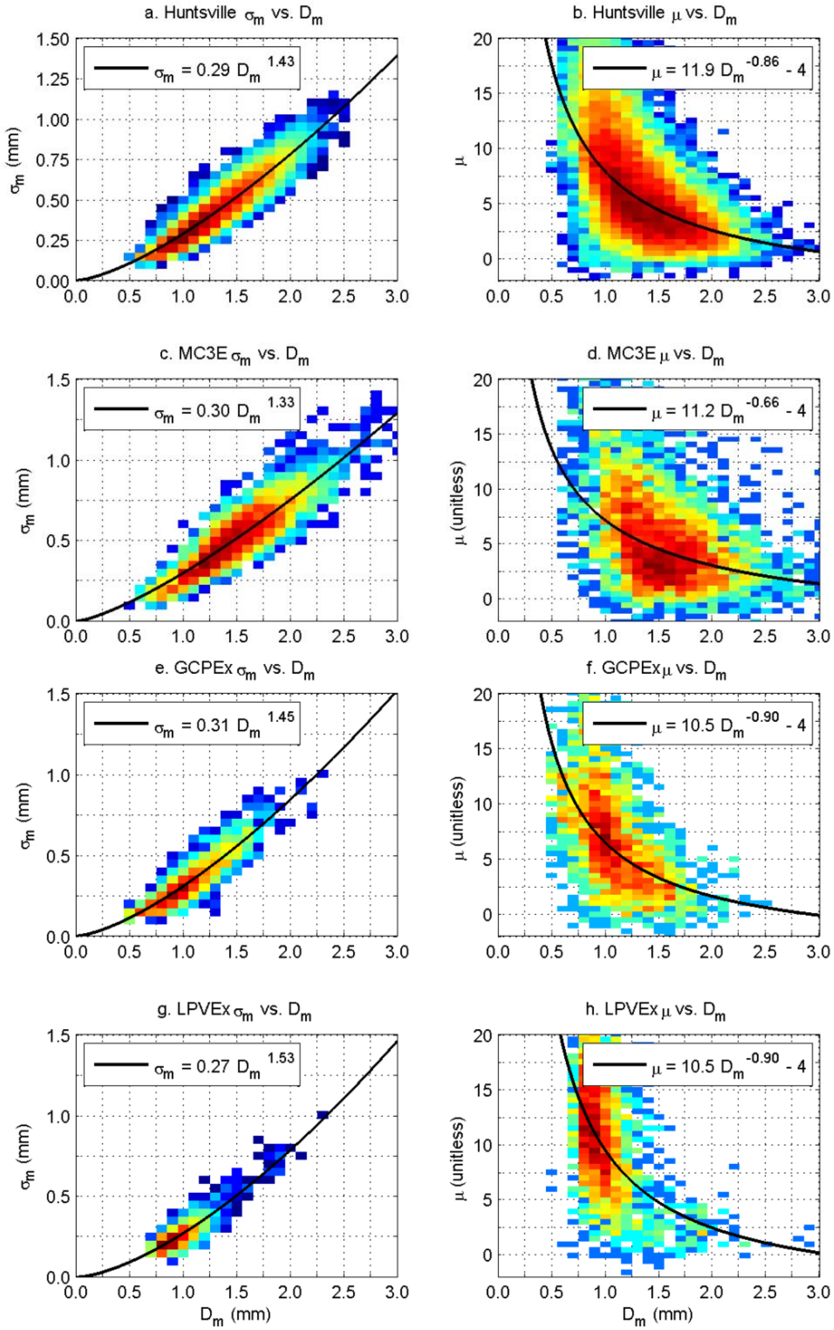


Normalized PDF of σ_y



74% of observations are within +/- 1 STD (a normal distribution would have 68%)

Observed σ_m vs. D_m | Calculated μ vs. D_m



Huntsville:, 20,954 samples

$\sigma_m = 0.29D_m^{1.43}$

MC3E: 5,175 samples

$\sigma_m = 0.30D_m^{1.33}$

GCPEx: 2,218 samples

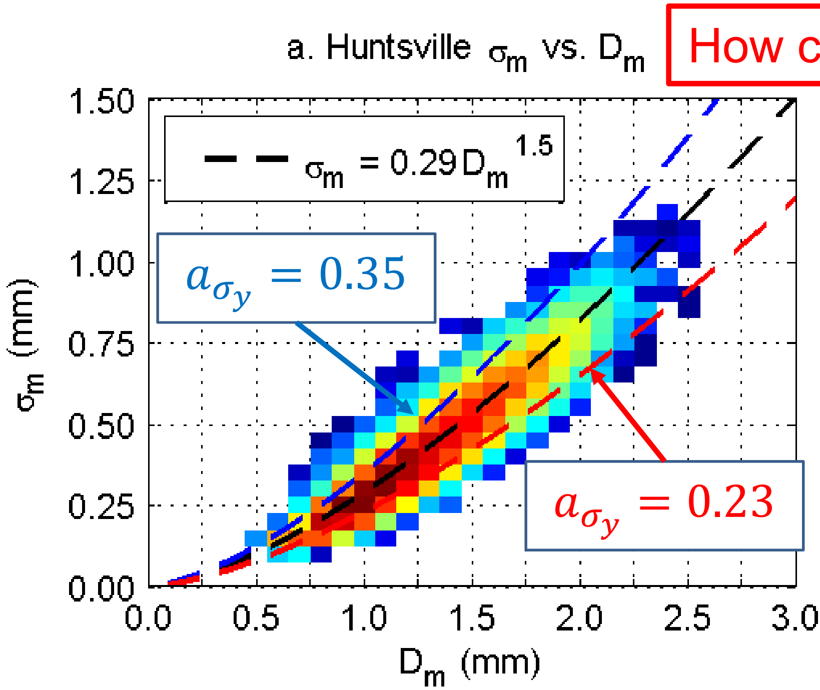
$\sigma_m = 0.31D_m^{1.45}$

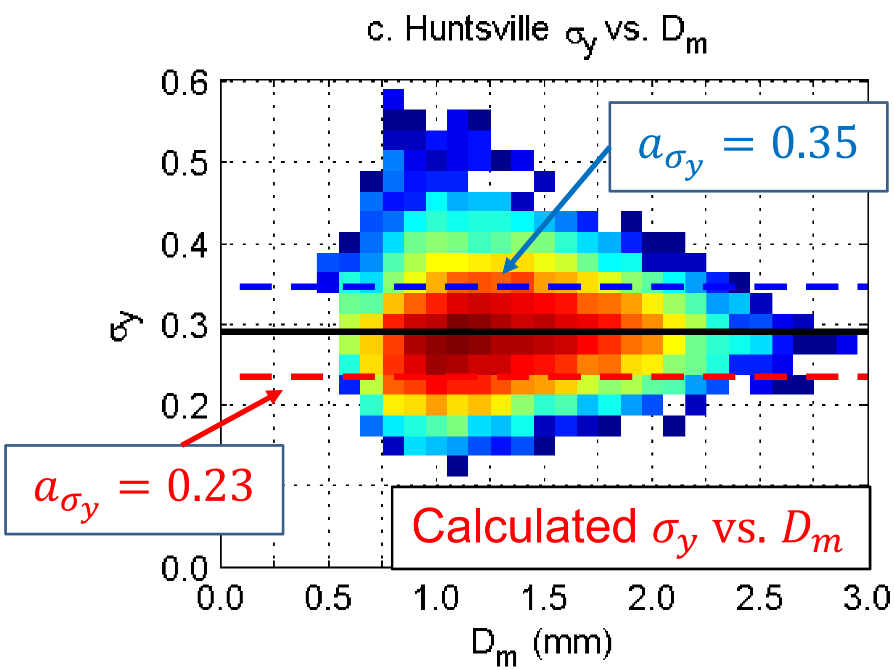
LPVEx: 2,454 samples

$\sigma_m = 0.27D_m^{1.53}$

Ensemble: 29,555 samples

$\sigma_m = 0.29D_m^{1.42}$





How can we get these constraints into Algorithms?

Adaptive Power-law Constraints for $\sigma_m - D_m$ and $\mu - D_m$

Observed b ranged from 1.33 to 1.53.

By setting $b = 1.5$,

$$_{constraint}\,\sigma_m = a_{\sigma_y} D_m^{1.5}$$

- Constraint is only a function of a_{σ_y}
- $\mu - D_m$ constraint has a simple form:

$$_{constraint}\,\mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$$

Change a_{σ_y} to get a different constraint.

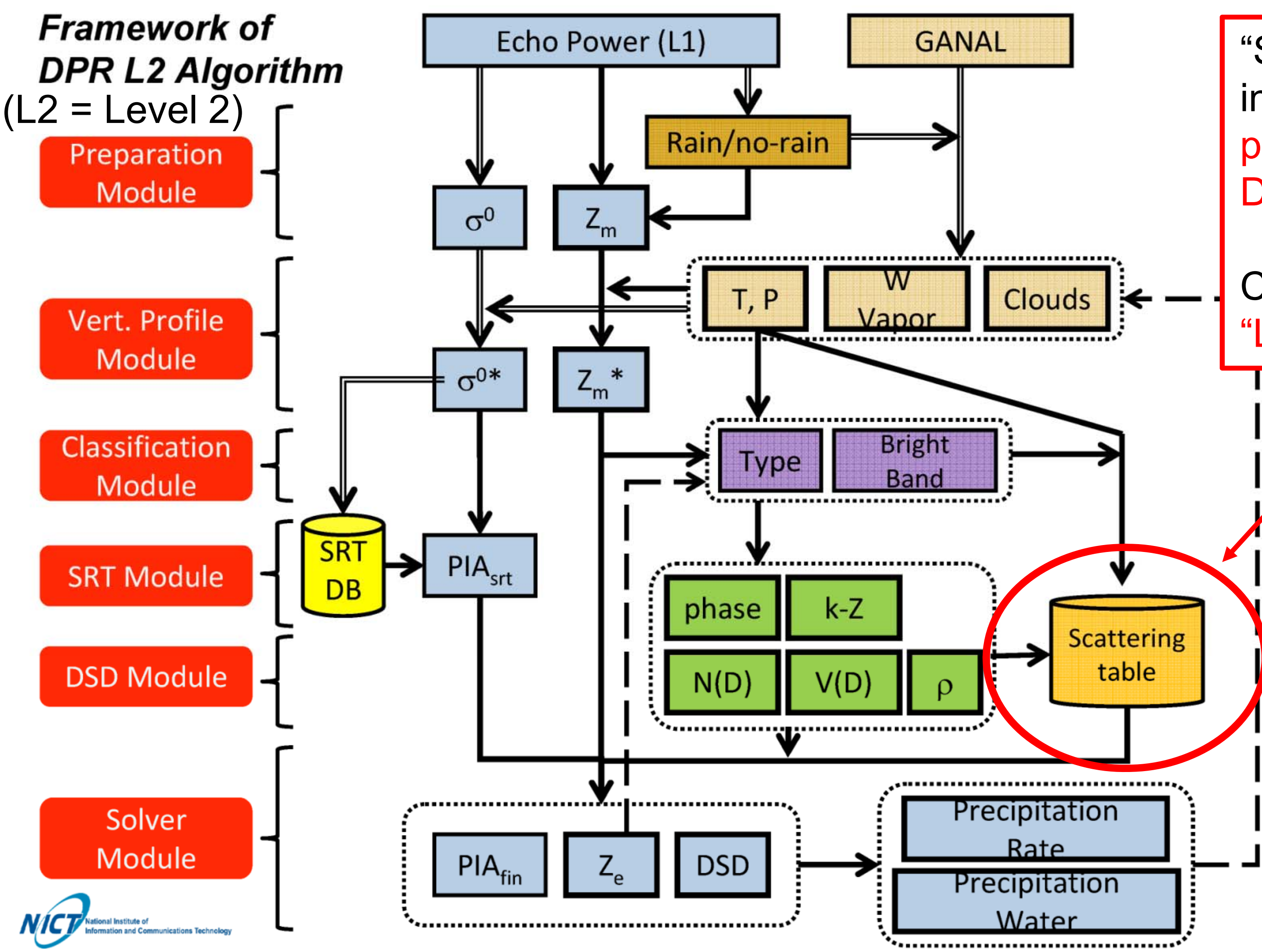
$_{constraint}\sigma_m = 0.35 D_m^{1.5} \Rightarrow \overline{\sigma_y} + std(\sigma_y)$

$_{constraint}\sigma_m = 0.29 D_m^{1.5} \Rightarrow \overline{\sigma_y}$ (best fit)

$_{constraint}\sigma_m = 0.23 D_m^{1.5} \Rightarrow \overline{\sigma_y} - std(\sigma_y)$

Developing a Framework to Incorporate GV Findings into Algorithms

For speed and efficiency, algorithms use Look-up Tables that include both Particle Habit and Particle Size Distribution.

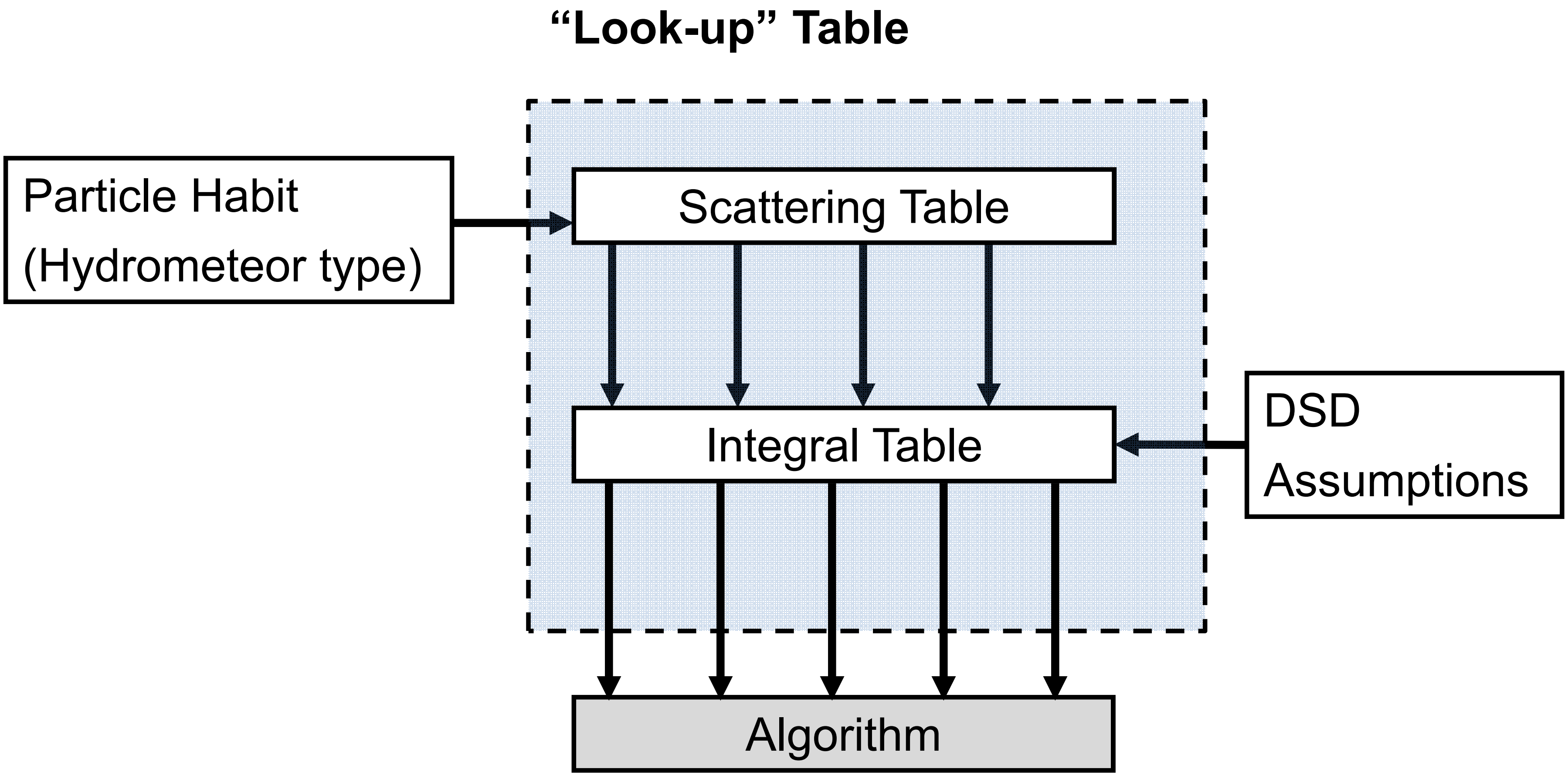


"Scattering Table" includes particle habit and DSD assumptions

Call this table a "Look-up Table"

From: Toshio Iguchi (NITC) and Shinta Seto (Univ. of Tokyo)
5th International Workshop for GPM Ground Validation, 10-12 July 2012, Toronto, Canada

Developing a Framework to Incorporate GV Findings into Algorithms



Bridging GV & Algorithms: Scattering Tables and Integral Tables

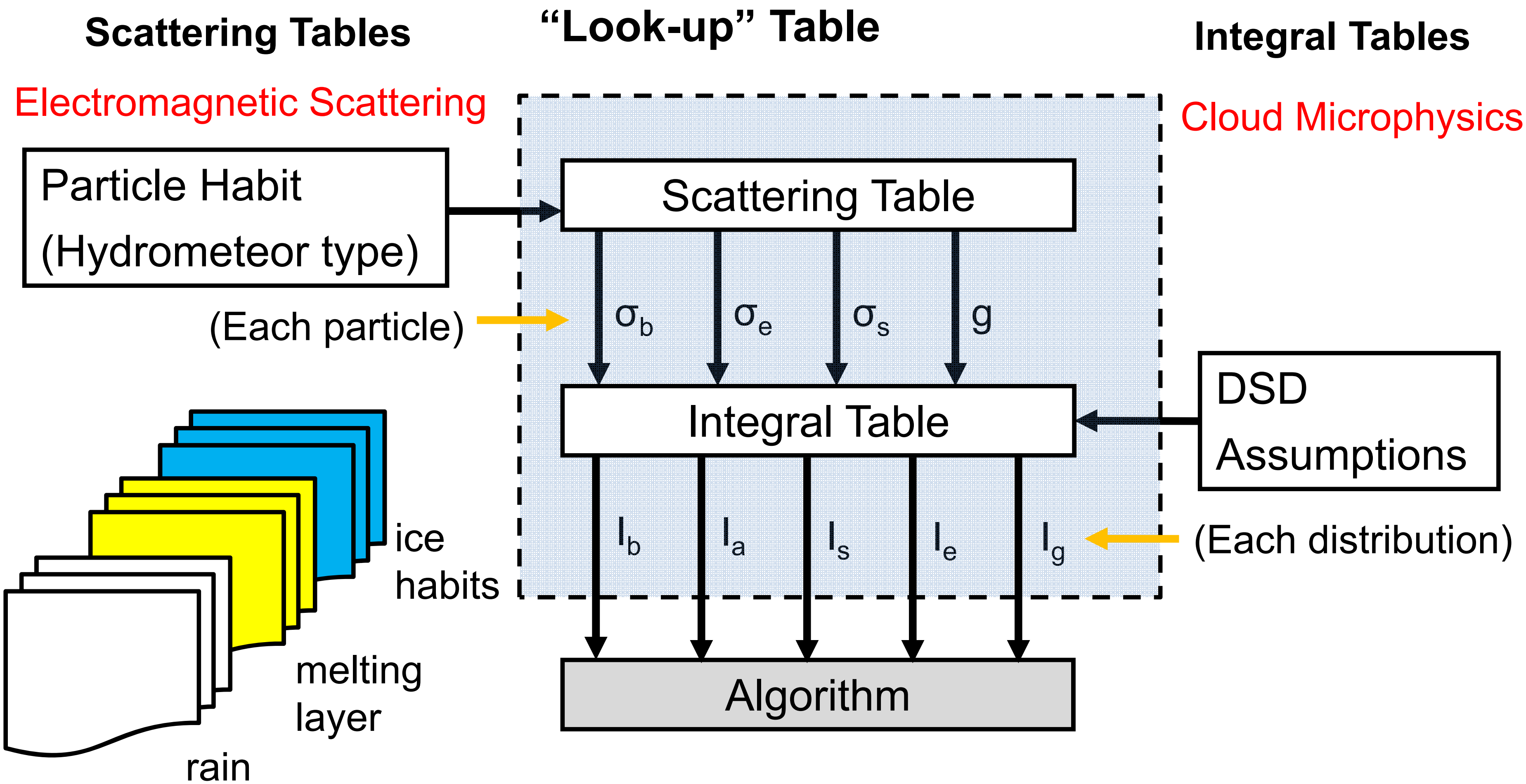
Scattering Tables

- Research: Electromagnetic Scattering
  - Interaction of electromagnetic waves with individual particles
- Vocabulary
  - Mie scattering
  - T-matrix
  - Incidence angle
  - Operating frequency
  - a/b relationship
  - Volume/mass relationship
  - Density
- Table Outputs
  - Backscattering cross section, σ_b
  - Extinction cross section , σ_e
  - Scattering cross section, σ_s
  - Asymmetry factor, g

Integral Tables

- Research: Cloud Microphysics
  - Integral quantities due to the distribution of particles
- Vocabulary
  - DSD assumption
  - N_w, D_m, μ parameters
  - μ-Λ relationship
  - σ_m-D_m relationship
  - Z, R, LWC
  - Z at Ku or Z at Ka
  - N_w-D_m or N_t-D_0 formulation
- Table Outputs
  - Normalized reflectivity coef., I_b
  - Normalized attenuation coef., I_a
  - Normalized scattering coef., I_s
  - Normalized emission coef., I_e
  - Normalized asymmetry coef., I_g

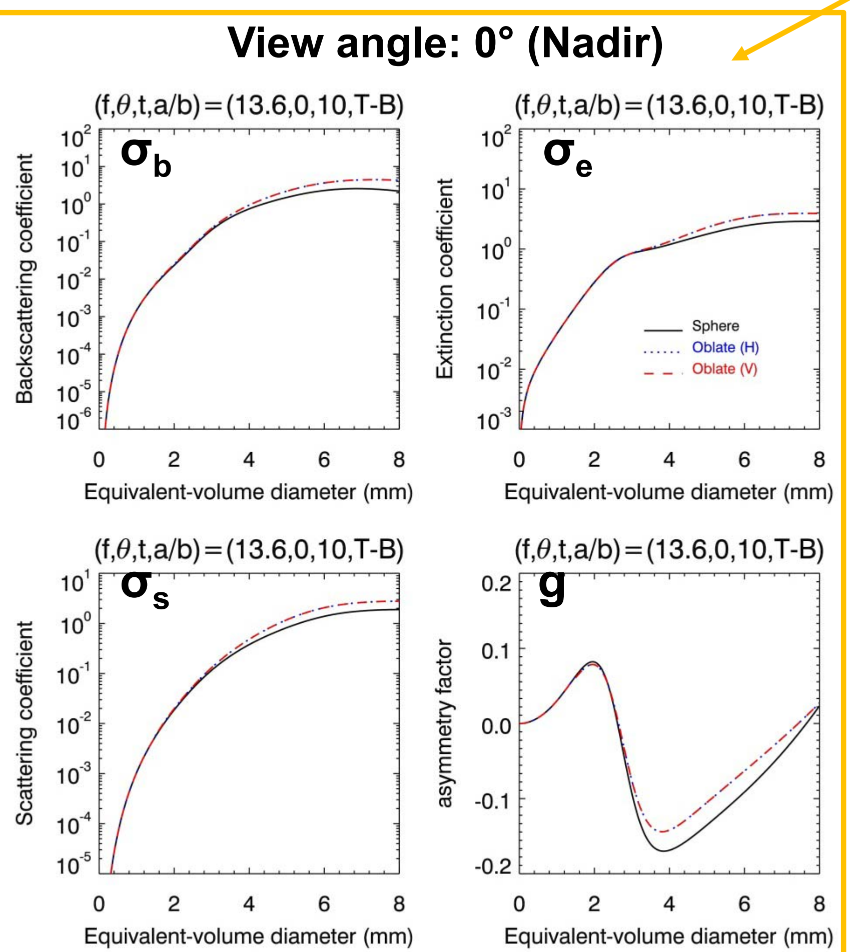
Objective D: Developing a Framework to Incorporate GV Findings into Algorithms



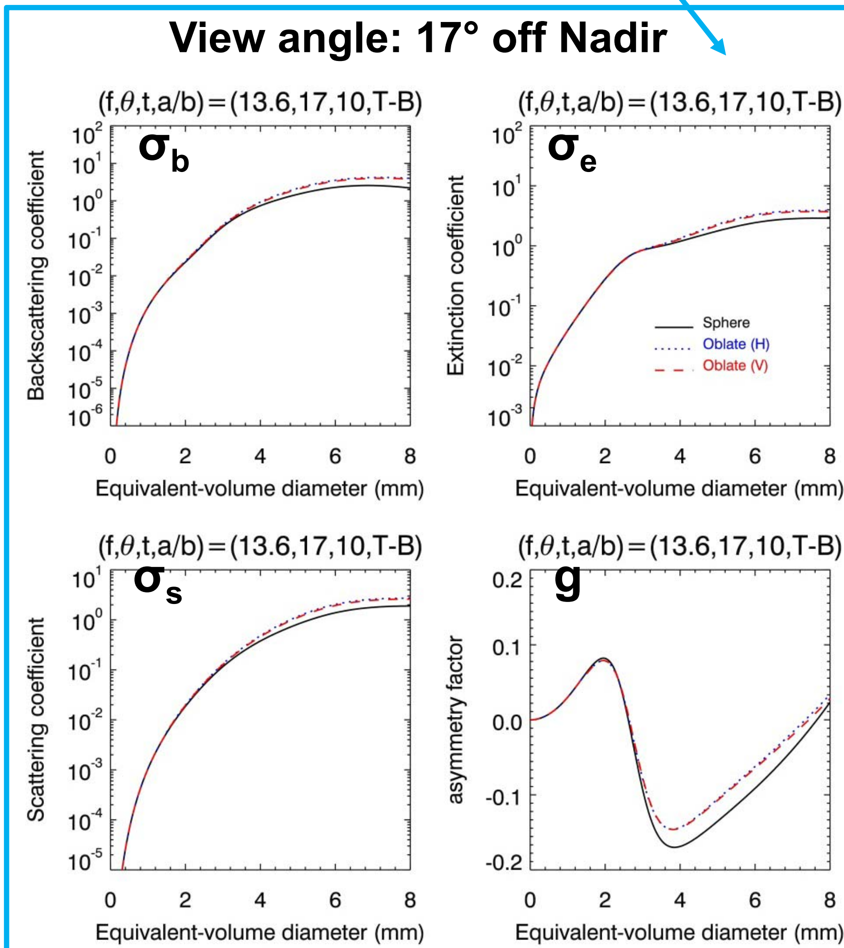
DSD WG Proof-of-Concept Project – Example Scattering Tables

13.6 GHz Radar:
7 Scattering Tables

a/b Relationship	0° (Nadir)	8.5° Off Nadir	17° Off Nadir
sphere	Yes	-	-
Beard-Chung	Yes	Yes	Yes
Thurai-Bringi	Yes	Yes	Yes



Temperature = 10 C



Calculations made by Liang Liao

Normalized Reflectivity Integral Tables

From Robert Meneghini

Given a DSD of the form:

$$N(D) = N_w f(\mu)\left(\frac{D}{D_m}\right)^{\mu} \exp\left(-\frac{(4+\mu)}{D_m}D\right)$$

What is the reflectivity given N(D) and a radar operating at wavelength λ?

$$Z(\lambda; N_w, D_m, \mu) = 10log\left(\frac{\lambda^4}{\pi^5|K_w|^2}\int_{D_{min}}^{D_{max}} N_w f(\mu)\left(\frac{D}{D_m}\right)^{\mu} exp\left(-\frac{(4+\mu)}{D_m}D\right)\sigma_b(\lambda, D)\, dD\right)$$

(3 parameters)

Backscattering cross section

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(3 parameters)

Backscattering cross section

N_w is a constant in the integral, so normalize such that N_w= 1:

$$I_b\,(\lambda; D_m, \mu) = 10log\left(\frac{\lambda^4}{\pi^5|K_w|^2}\int_{D_{min}}^{D_{max}} f(\mu)\left(\frac{D}{D_m}\right)^{\mu} exp\left(-\frac{(4+\mu)}{D_m}D\right)\sigma_b(\lambda, D)\, dD\right)$$

(2 parameters)

Normalized Reflectivity Integral Tables

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(3 parameters)

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(2 parameters)

Select a μ constraint (μ = constant, μ = f(D_m)). For example, if μ=3:

$$I_b(\lambda; D_m)\Big|_{\mu=3} = 10log\left(\frac{\lambda^4}{\pi^5 |K_w|^2} \int_{D_{min}}^{D_{max}} f(3) \left(\frac{D}{D_m} \right)^{3} exp\left(-\frac{(4+3)}{D_m} D \right) \sigma_b(\lambda, D)\, dD \right)$$

(1 parameter)

After setting a μ constraint, I_b is only a function of D_m

Summarizing Key Features of Integral Tables

- Inside each Integral Table:
  1. Tables are normalized such that $N_w = 1$
  2. Single μ assumption: μ = constant or $_{constraint}\ \mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$
  3. Tables indexed using D_m (1-dimensional look-up table)
- Outside of Integral Table:
  1. N_w is scaled by the algorithm
  2. Attenuation correction is performed by the algorithm
  3. Algorithm selects different integral tables to get different μ assumptions as it converges

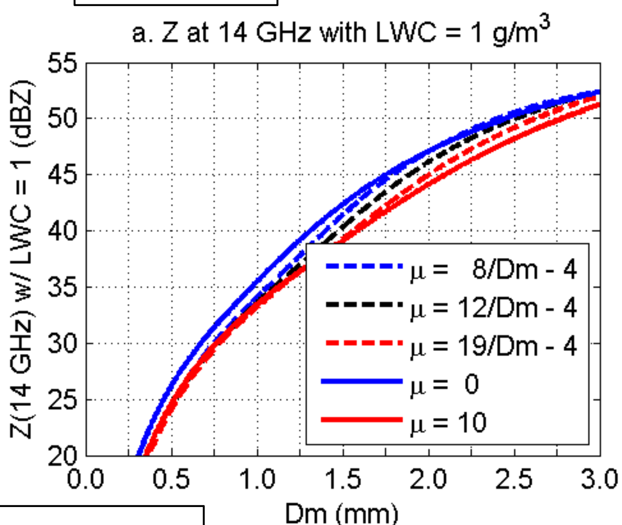
Simple scaling from I_b & I_a to Z & k:

$$Z(\lambda; N_w, D_m)\Big|_{\mu=3} = 10\log(N_w) + I_b(\lambda; D_m)\Big|_{\mu=3}$$

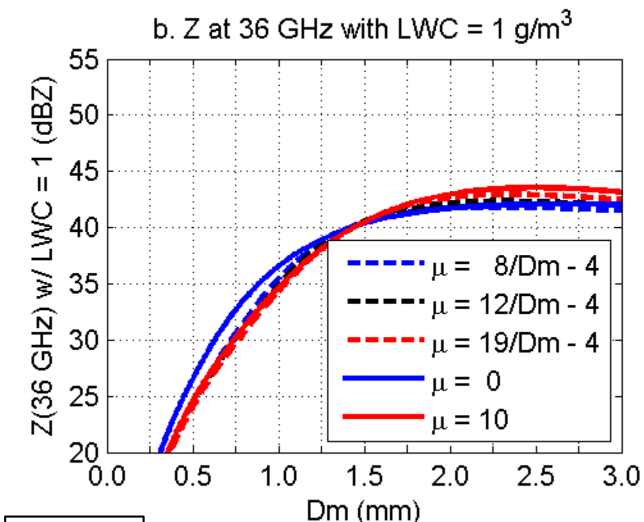
$$k(\lambda; N_w, D_m)\Big|_{\mu=3} = N_w I_a(\lambda; D_m)\Big|_{\mu=3}$$

Reflectivity Integral Tables at 14 and 36 GHz Normalized so that LWC = 1 g/m^3

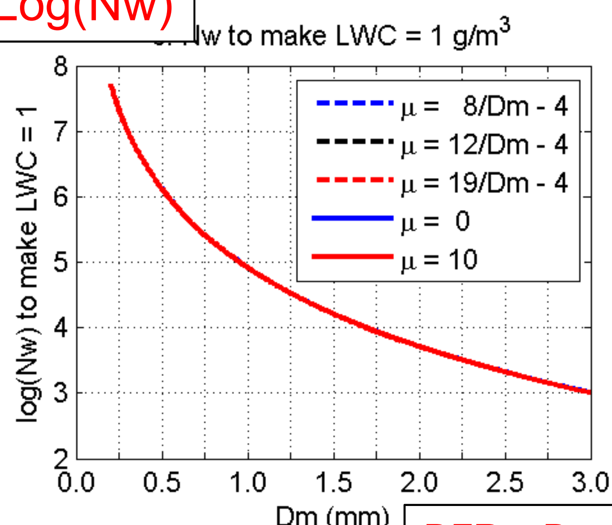
14 GHz



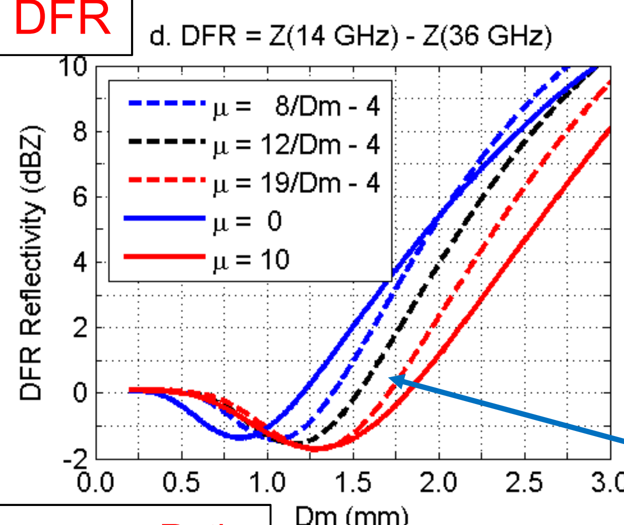
36 GHz



Log(Nw)



DFR



DFR = Dual Frequency Ratio

Constraint:

$$\mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$$

with:

$$a_{\sigma_y} = 0.23, 0.29, 0.35$$

Which are approx:

$$\mu \sim \frac{19}{D_m} - 4$$ Narrower

$$\mu \sim \frac{12}{D_m} - 4$$ Centerline

$$\mu \sim \frac{8}{D_m} - 4$$ Broader

74% of 2DVD obs are within +/- 1 STD (blue & red dash)

Radar Integral Tables

Adaptive power-law constraint being tested in combined algorithm.
From Joe Munchak

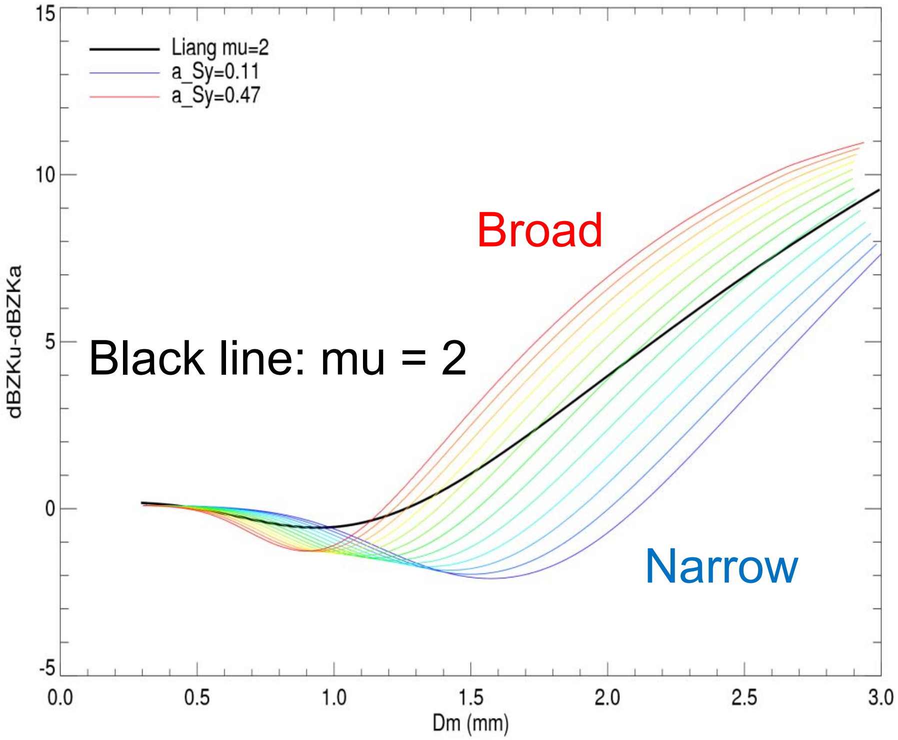
$$\mathit{constraint}\ \sigma_m = a_{\sigma_y} D_m^{1.5}$$

$$\mathit{constraint}\ \mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$$

a_{σ_y} ranged from 0.11 to 0.47
(very narrow to very broad)

(Limits of integration = 3*Dm)
(Spherical drops)

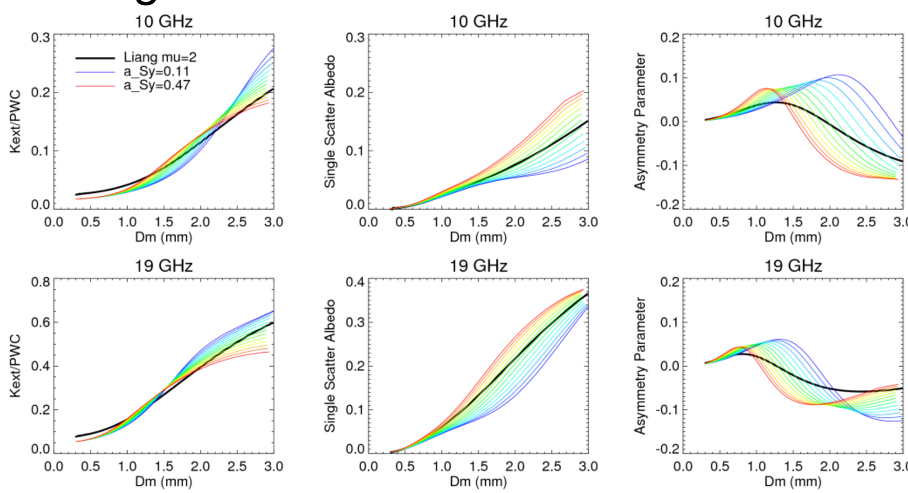
DFR = Dual Frequency Ratio
DWR = Dual Wavelength Ratio

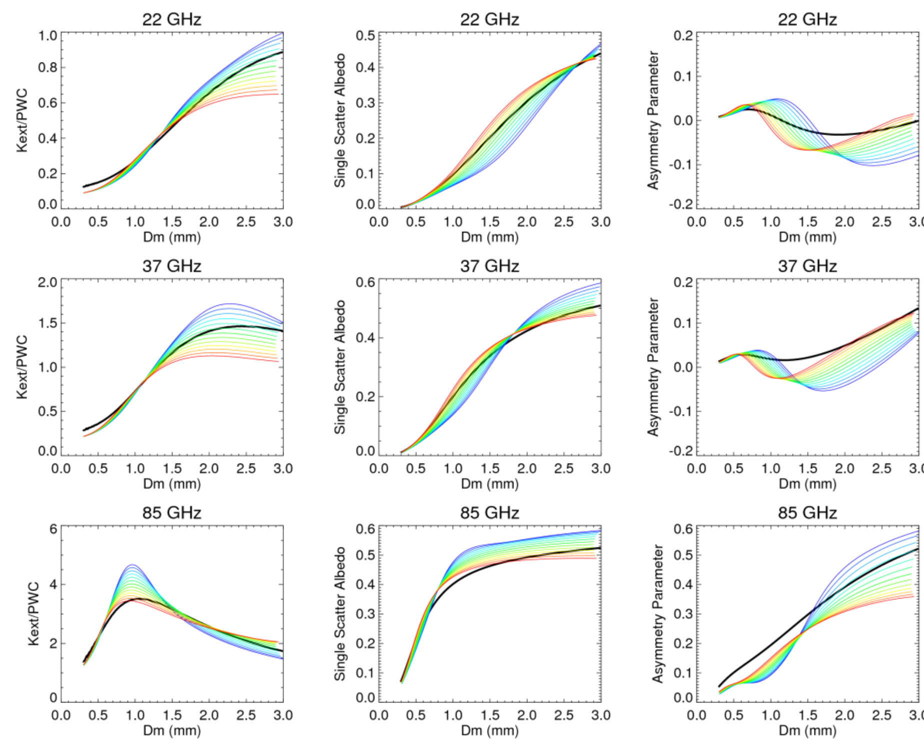


Radiometer Integral Tables

Adaptive power-law constraint being tested in combined algorithm.
From Joe Munchak

Parameters needed for radiative transfer calculations are also integrated over the same DSD with the same code to ensure consistency between radar and radiometer algorithm modules.

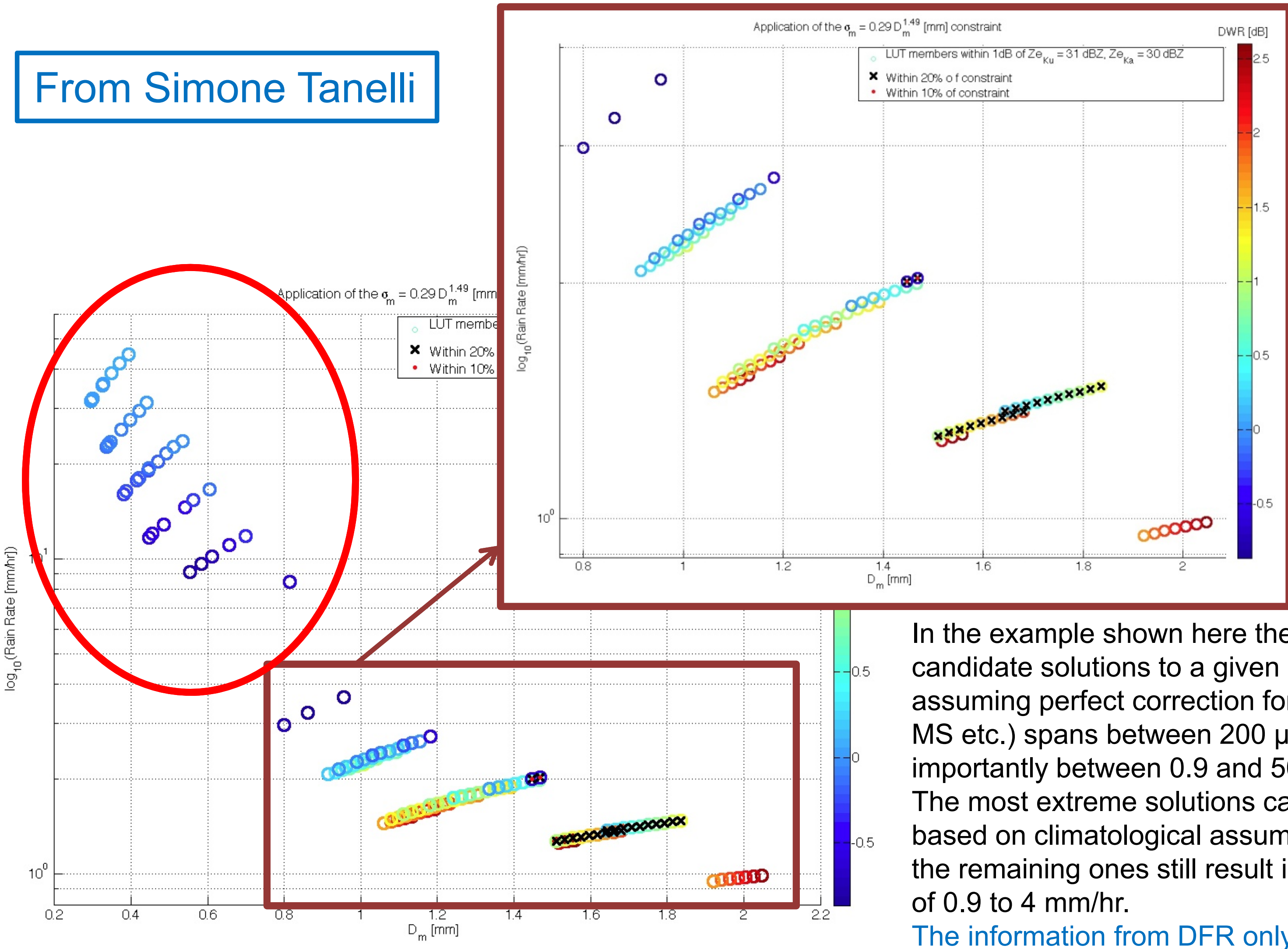




Narrow (a_{σ_y} = 0.11) Broad (a_{σ_y} = 0.47)

Inclusion of the σ_m-D_m constraint allows to significantly reduce the search domain for the APR-2 Bayesian retriever that uses a pre-generated Look-Up Table. Furthermore, the reduction is often almost orthogonal to the DFR information.

From Simone Tanelli



In the example shown here the ‘initial’ population of candidate solutions to a given Ku/Ka Ze pair (note: assuming perfect correction for attenuation, no NUBF, no MS etc.) spans between 200 μm and 2 mm and most importantly between 0.9 and 50 mm/hr.
The most extreme solutions can be easily discarded based on climatological assumptions (red ellipse), but the remaining ones still result in an intrinsic uncertainty of 0.9 to 4 mm/hr.
The information from DFR only reduces the range in half (assuming a 0.5 dB accuracy). Combining the DFR with the σ_m-D_m constraint reduces the range by one order of magnitude (black x and red dots).

Impact of this constraint is being quantitatively evaluated on the GCPEX rain retrievals

Concluding Remarks (1/2)

Develop physically based relationships between DSD parameters

- NASA GPM DSD Working Group is investigating relationships between DSD parameters to address assumptions used in retrieval algorithms.
- $\sigma_m \sim D_m^{1.5}$ relationship appears robust & observed in several field campaigns.
- Defined an adaptive constraint with one parameter: $\mu = \frac{1}{a_{\sigma_y}^2 D_m} - 4$
- Williams et al, 2013: Adaptive Raindrop Size Distribution Constraint for Probabilistic Rainfall Retrieval Algorithms, *submitted to J. Appl. Meteor. Climatol*.

Develop a framework to incorporate GV findings into Algorithms

- Divide Algorithm "Look-up Tables" into Scattering and Integral Tables.
- Scattering Tables describe the electromagnetic properties of particles
- Integral Tables describe particle size distributions

Benefits of dividing Look-up Tables into Scattering and Integral Tables:

1. Researchers can work independently – Developing scattering tables is independent of investigating particle size distributions.
2. Provides a framework to incorporate GV findings into Look-up Tables used by satellite algorithms.
3. Provides a communication framework for particle scattering modelers, observational scientists, and algorithm developers.

Concluding Remarks (2/2)

Next Steps for the DSD Working Group

- We've made great progress on DSD parameter relationships
  - …more work is still needed…
  - How do convective / stratiform rain regimes map into $\sigma_m - D_m$ relationships?
  - Is there any relationship between σ_y and N_w?
  - Does D_{max} play a role in the power-law relationships?

Change the structure of our monthly Teleconference calls

- Rotate through different facilitators
  - Christopher Williams – DSD parameter relationships (same as before)
  - Steve Nesbitt – building the column and moving to aircraft observations
  - Joe Munchak – algorithm prospective and issues with ice & frozen PSDs

If you want to be involved with the DSD Working Group, send email:

christopher.williams@colorado.edu

Members (to) and Friends (cc)