OBJECTIVE CHARACTERIZATION OF RAIN MICROPHYSICS

Francisco J. Tapiador

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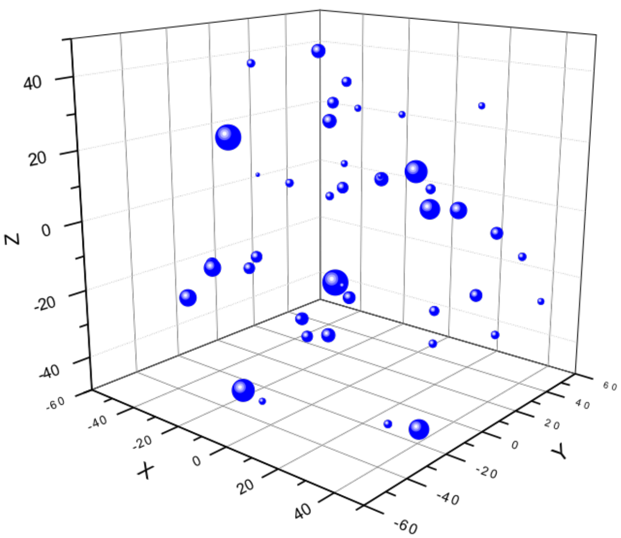
Motivation

GCMs, RCMs, CRMs and NWP models diagnose/prognose

$$\frac{\partial N_T}{\partial t} \qquad \frac{\partial q_h}{\partial t}$$

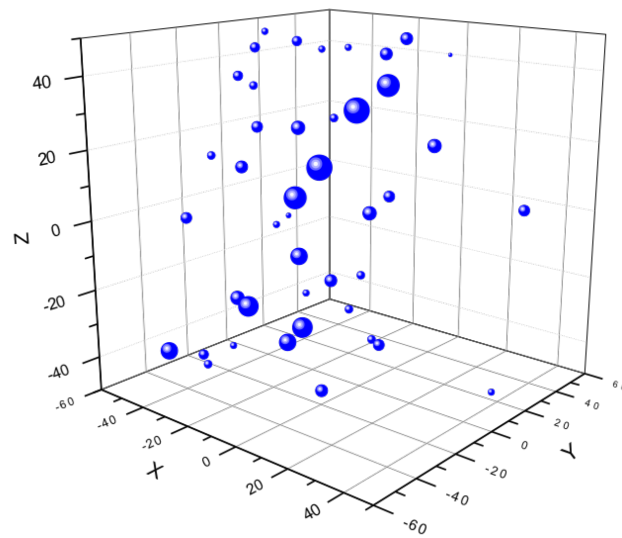
So it is important to retrieve these quantities independently to develop, calibrate, tune, verify and validate those models

microphysics (mp)



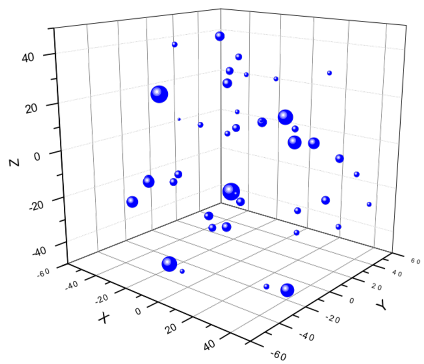
t_1

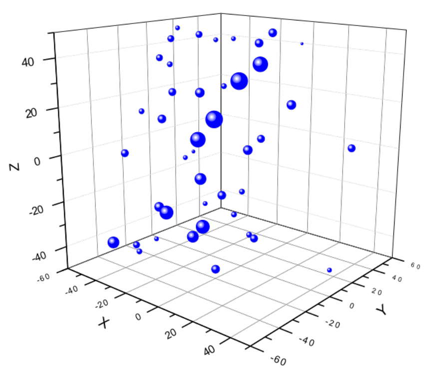
$$\left\{ \begin{array}{c} P \\ q \\ T \\ w, K \\ \theta \end{array} \right\}$$



t_2

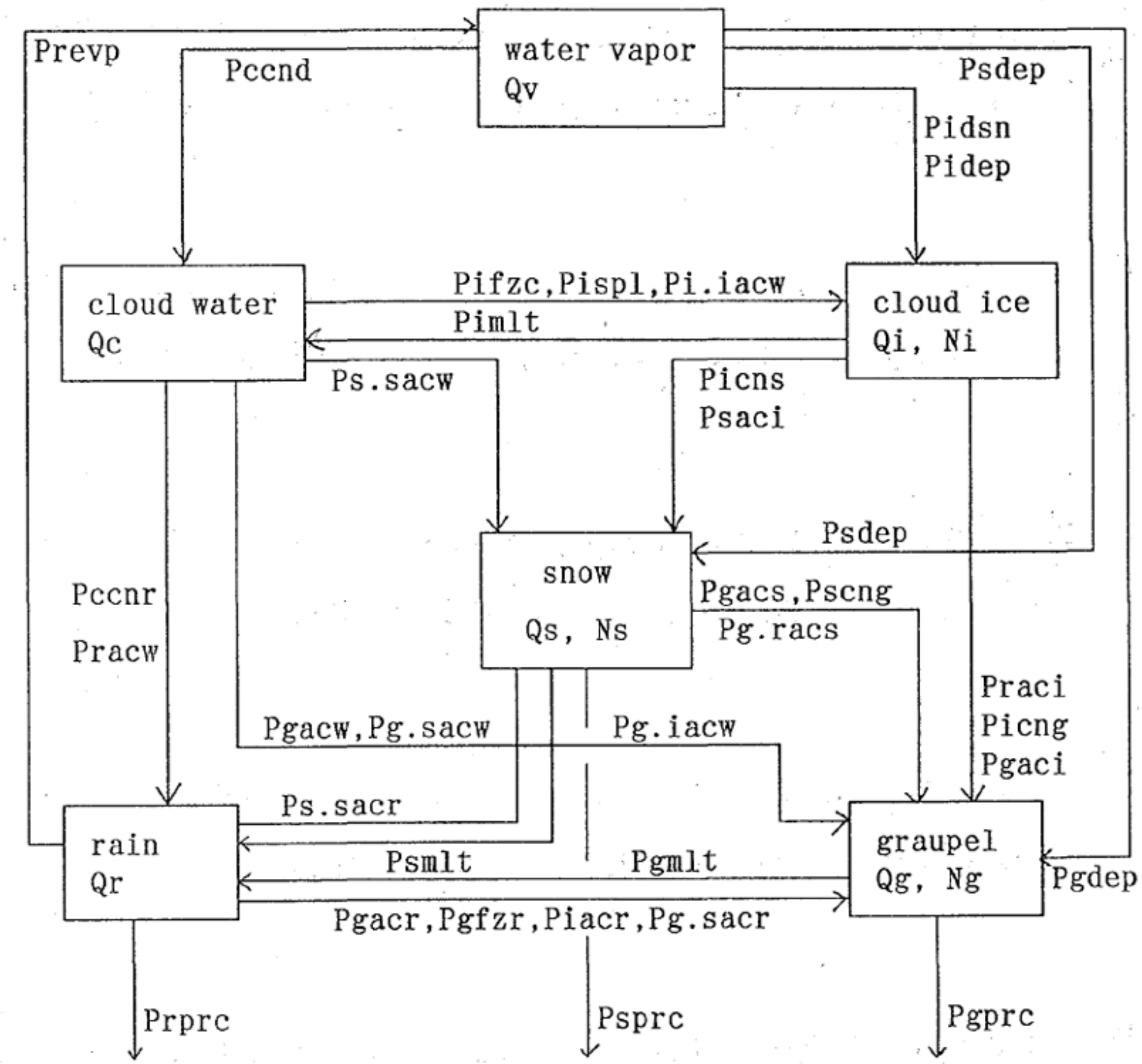
for instance, in practice, in the P3 mp





$$N = \int_0^\infty N'(D)\, dD = \int_0^\infty N_0 D^\mu e^{-\lambda D}\, dD \quad \text{and}$$

$$q = \int_0^\infty m(D) N'(D)\, dD = \int_0^\infty m(D) N_0 D^\mu e^{-\lambda D}\, dD$$

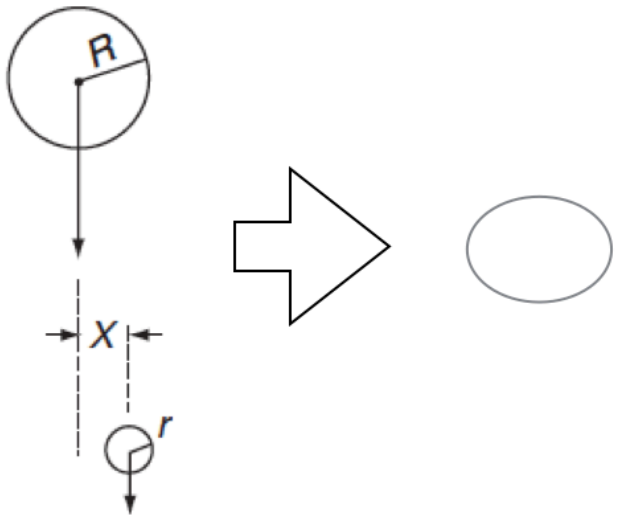


Ikawa&Saito 1991

Rain aggregation

PRaggR $(\partial\{N_R, q_R\}/\partial t)_{RaggR}$ 1/41

Pictorial representation



Assumptions

- Spherical drops
- Fall at terminal v
- Spaced in the vertical
- Calm, laminar flux
- Large efficiency in collision and coalescence (=1)

Effects on N and gamma pdf

$N_V =$	$m_V =$	$s_V =$
$N_R \downarrow$	$m_R \uparrow$	$s_R \downarrow$
$N_S =$	$m_S =$	$s_S =$
$N_I =$	$m_I =$	$s_I =$
$N_G =$	$m_G =$	$s_G =$

Parameterizations

Beheng (1994)

$(\partial N_R/\partial t)_{aggRR} = -8.0 \cdot 10^3 \cdot N_R \cdot q_R$

$(\partial q_R/\partial t)_{aggRR} = 0$

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!.............................................................................
! SELF-COLLECTION OF RAIN DROPS
! FROM BEHENG(1994)
! FROM NUMERICAL SIMULATION OF THE STOCHASTIC COLLECTION EQUATION
! AS DESCRINED ABOVE FOR AUTOCONVERSION

         IF (QR3D(K).GE.1.E-8) THEN
! include breakup add 10/09/09
            dum1=300.e-6
            if (1./lamr(k).lt.dum1) then
            dum=1.
            else if (1./lamr(k).ge.dum1) then
            dum=2.-exp(2300.*(1./lamr(k)-dum1))
            end if
!            NRAGG(K) = -8.*NR3D(K)*QR3D(K)*RHO(K)
            NRAGG(K) = -5.78*dum*NR3D(K)*QR3D(K)*RHO(K)
         END IF
```

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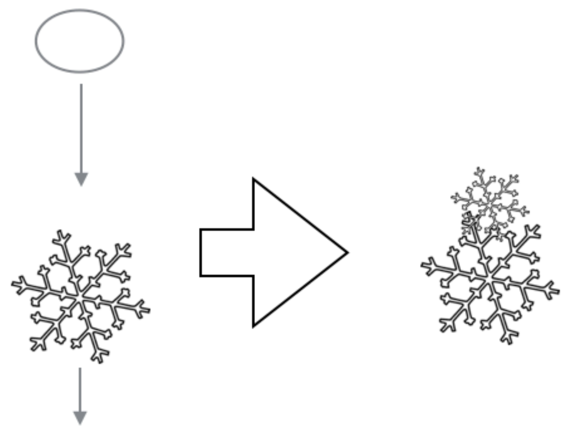
Accretion of snow by rain water

PSacR $(\partial\{N_R,q_R\}/\partial t)_{SacR}$ 2/41

Pictorial representation



Assumptions

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$N_R \downarrow$	$m_R \uparrow$	$s_R \downarrow$
$N_S =$	$m_S \uparrow$	$s_S \uparrow$
$N_I =$	$m_I =$	$s_I =$
$N_G =$	$m_G =$	$s_G =$

Parameterizations

Morrison's mp, based on Ikawa and Saito (1991)

```
!.......................................................................
! ACCRETION OF RAIN WATER BY SNOW
! FORMULA FROM IKAWA AND SAITO, 1991, USED BY REISNER ET AL, 1998

         IF (QR3D(K).GE.1.E-8.AND.QNI3D(K).GE.1.E-8) THEN

            UMS = ASN(K)*CONS3/(LAMS(K)**BS)
            UMR = ARN(K)*CONS4/(LAMR(K)**BR)
            UNS = ASN(K)*CONS5/LAMS(K)**BS
            UNR = ARN(K)*CONS6/LAMR(K)**BR

! SET REASLISTIC LIMITS ON FALLSPEEDS

! bug fix, 10/08/09
            dum=(rhosu/rho(k))**0.54
            UMS=MIN(UMS,1.2*dum)
            UNS=MIN(UNS,1.2*dum)
            UMR=MIN(UMR,9.1*dum)
            UNR=MIN(UNR,9.1*dum)

            PRACS(K) = CONS41*(((1.2*UMR-0.95*UMS)**2+                   &
                  0.08*UMS*UMR)**0.5*RHO(K)*                        &
                  N0RR(K)*N0S(K)/LAMR(K)**3*                           &
                  (5./(LAMR(K)**3*LAMS(K))+                      &
                  2./(LAMR(K)**2*LAMS(K)**2)+                    &
                  0.5/(LAMR(k)*LAMS(k)**3)))

            NPRACS(K) = CONS32*RHO(K)*(1.7*(UNR-UNS)**2+              &
                0.3*UNR*UNS)**0.5*N0RR(K)*N0S(K)*                 &
                (1./(LAMR(K)**3*LAMS(K))+                         &
                 1./(LAMR(K)**2*LAMS(K)**2)+                      &
                 1./(LAMR(K)*LAMS(K)**3))

! MAKE SURE PRACS DOESN'T EXCEED TOTAL RAIN MIXING RATIO
! AS THIS MAY OTHERWISE RESULT IN TOO MUCH TRANSFER OF WATER DURING
! RIME-SPLINTERING

            PRACS(K) = MIN(PRACS(K),QR3D(K)/DT)

! COLLECTION OF SNOW BY RAIN - NEEDED FOR GRAUPEL CONVERSION CALCULATIONS
! ONLY CALCULATE IF SNOW AND RAIN MIXING RATIOS EXCEED 0.1 G/KG

! HM MODIFY FOR WRFV3.1
!             IF (IHAIL.EQ.0) THEN
            IF (QNI3D(K).GE.0.1E-3.AND.QR3D(K).GE.0.1E-3) THEN
            PSACR(K) = CONS31*(((1.2*UMR-0.95*UMS)**2+                  &
                  0.08*UMS*UMR)**0.5*RHO(K)*                       &
                 N0RR(K)*N0S(K)/LAMS(K)**3*                              &
                  (5./(LAMS(K)**3*LAMR(K))+                      &
                  2./(LAMS(K)**2*LAMR(K)**2)+                    &
                  0.5/(LAMS(K)*LAMR(K)**3)))
            END IF
!             END IF

         END IF
```

Likewise the collection of snow by rain water (IS, Eq. (58)) is given as,

$$P_{sacr} = \pi^2 E_{sr} \sqrt{(\alpha \hat{U}_r - \beta \hat{U}_s)^2 + \gamma \hat{U}_r \hat{U}_s} \frac{\rho_s}{\rho} N_{sr} N_{ss} \left(\frac{5}{\lambda_s^3 \lambda_r} + \frac{2}{\lambda_s^2 \lambda_r^2} + \frac{0.5}{\lambda_s \lambda_r^3} \right). \quad (A.48)$$

The number of collisions between snow and rain water (IS, Eq. (60)) is given as,

$$N_{sacr} = N_{racs} = \frac{\pi}{2} E_{sr} \sqrt{\alpha (\hat{U}_{nr} - \hat{U}_{ns})^2 + \beta \hat{U}_{nr} \hat{U}_{ns}} N_{sr} N_{ss} \left(\frac{1}{\lambda_r^3 \lambda_s} + \frac{1}{\lambda_r^2 \lambda_s^2} + \frac{1}{\lambda_r \lambda_s^3} \right). \quad (A.49)$$

where $\alpha = 1.7$, and $\beta = 0.3$.

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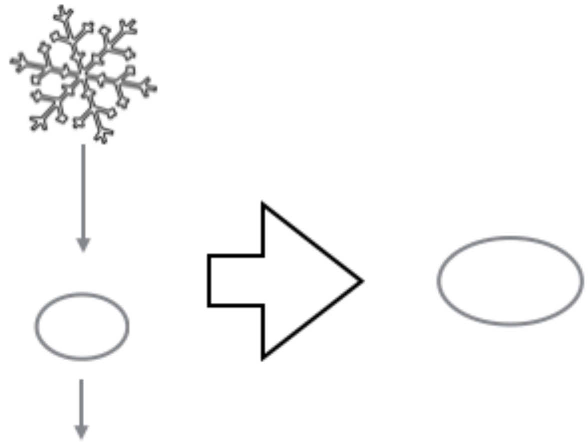
Accretion of rain water by snow

PRacS

$(\partial\{N_R, q_R\}/\partial t)_{RacS}$

Pictorial representation



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$N_S \downarrow$	m_S ?	s_S ?
$N_I =$	$m_I =$	$s_I =$
$N_G =$	$m_G =$	$s_G =$

Parameterizations

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At the end, the evolution is linked with the initial DSD model

$\partial N_h / \partial t$ $\partial q_h / \partial t$

Drop number

Drop number +
Shape of the RDSD

RDSD models

The N_0 model

$$n(D) = N_0 D^{\mu} exp(-\Lambda D)$$

[N_0 model]

N_0 model - issues

$$n(D) = N_0 D^{\mu} exp(-\Lambda D)$$

[N_0 model]

- Such expression is not a PDF
- N_0 has units of $m^{-4-\mu}$
- Parameters are not independent

N_0 model - issues

$$n(D) = N_0 D^{\mu} exp(-\Lambda D)$$

$$N_0 = N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)}$$

N_0 model - issues

$$n(D) = N_0 D^{\mu} exp(-\Lambda D)$$

$$N_0 = N_T \frac{\Lambda^{\mu+1}}{\Gamma(\mu+1)}$$

N_0 model - issues

$$n(D) = N_0 D^{\mu} exp(-\Lambda D)$$

- Not only mathematically ill-defined
- Also disconnected from mp modeling

N_w , normalized model

$$n(D) = N_w f_N(\mu) \left(\frac{D}{D_m}\right)^{\mu} \exp\left[-(4+\mu)\frac{D}{D_m}\right]$$

$$f_N(\mu) = \frac{6}{4^4} \frac{(4+\mu)^{\mu+4}}{\Gamma(\mu+4)}$$

N_w model - issues

$$N_w = \frac{4^4}{\pi \rho_w}\left(\frac{10^3 W}{D_m^4}\right)$$

N_w model - issues

$$N_w = \frac{4^4}{\pi \rho_w}\left(\frac{10^3 W}{D_m^4}\right) \qquad \mathrm{N_w} = k\,\frac{W}{{D_m}^4}$$

$$W = \frac{\pi \rho_w 10^{-6}}{6}\int_{Dmin}^{Dmax} D^3\, n(D) dD \qquad \mathrm{D_m} = \frac{\int_{Dmin}^{Dmax} n(D) D^4\, dD}{\int_{Dmin}^{Dmax} n(D) D^3\, dD}$$

- Parameters are also intrinsically dependent
- Also disconnected from mp modeling

Alternative: N_T-based RDSDs

$$n(D) = N_T \cdot p(D)$$

Drop number

Shape of the RDSD
p(D) is a proper PDF: (and we need a PDF because we are doing statistics)

There is even a general form for the DSD as an (infinite) sum of exponentials:

$$n(D) = N_T \cdot p(D) = N_T \cdot \frac{\exp\left(-\lambda_0 - \lambda_1 f_1(D) - \lambda_2 f_2(D) - \cdots - \lambda_z f_z(D)\right)}{Z}$$

f_i can be any functions

for instance, central or raw moments

Alternative: N_T-based RDSDs

Maxent DSD

$$\begin{cases} E[D] = \int_{Dmin}^{Dmax} D \quad p(D)dD \\ E[D^2] = \int_{Dmin}^{Dmax} D^2 \, p(D)dD \end{cases}$$

$$n(D) = N_T \cdot p(D) = N_T \cdot \frac{\exp(-\lambda_0 - \lambda_1 D - \lambda_2 D^2)}{\sum_D \exp(-\lambda_0 - \lambda_1 D - \lambda_2 D^2)}$$

Alternative:
N_T-based RDSDs

Maxent gamma DSD

$$\begin{cases} E[D] = \int_{Dmin}^{Dmax} D \quad p(D)dD \\ E[log(D)] = \int_{Dmin}^{Dmax} \log(\mathrm{D})\, p(D)dD \end{cases}$$

$$\mathrm{n(D)} = \mathrm{N_T} \cdot \mathrm{p(D)} = \mathrm{N_T} \cdot \frac{\exp(-\lambda_0 - \lambda_1 \mathrm{D} - \lambda_2 \log(\mathrm{D}))}{\sum_{\mathrm{D}} \exp(-\lambda_0 - \lambda_1 \mathrm{D} - \lambda_2 \log(\mathrm{D}))}$$

Alternative: N_T-based RDSDs

$$n(D) = N_T \cdot \Lambda^{\mu+1} D^{\mu} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$$

[Maxent-Gamma model]

$$Z = k_1 N_T \Lambda^{-6} \frac{\gamma(\mu+7, D_{\max}\Lambda) - \gamma(\mu+7, D_{\min}\Lambda)}{\Gamma(\mu+1)}$$

$$W = k_2 N_T \Lambda^{-3} \frac{\gamma(\mu+4, D_{\max}\Lambda) - \gamma(\mu+4, D_{\min}\Lambda)}{\Gamma(\mu+1)}$$

$$R = k_3 v_1 N_T \Lambda^{-(3+v_2)} \frac{\gamma(\mu+4+v_2, D_{\max}\Lambda) - \gamma(\mu+4+v_2, D_{\min}\Lambda)}{\Gamma(\mu+1)}$$

$$D_m = \Lambda^{-1} \frac{\gamma(\mu+5, D_{\max}\Lambda) - \gamma(\mu+5, D_{\min}\Lambda)}{\gamma(\mu+4, D_{\max}\Lambda) - \gamma(\mu+4, D_{\min}\Lambda)}, \text{ does not}$$

depend on N_T

$$\sigma_m^2 = \Lambda^{-2} \frac{\gamma(\mu+6, D_{\max}\Lambda) - \gamma(\mu+6, D_{\min}\Lambda)}{\gamma(\mu+4, D_{\max}\Lambda) - \gamma(\mu+4, D_{\min}\Lambda)} - \left[\frac{\gamma(\mu+5, D_{\max}\Lambda) - \gamma(\mu+5, D_{\min}\Lambda)}{\gamma(\mu+4, D_{\max}\Lambda) - \gamma(\mu+4, D_{\min}\Lambda)}\right]^2, \text{ does not}$$

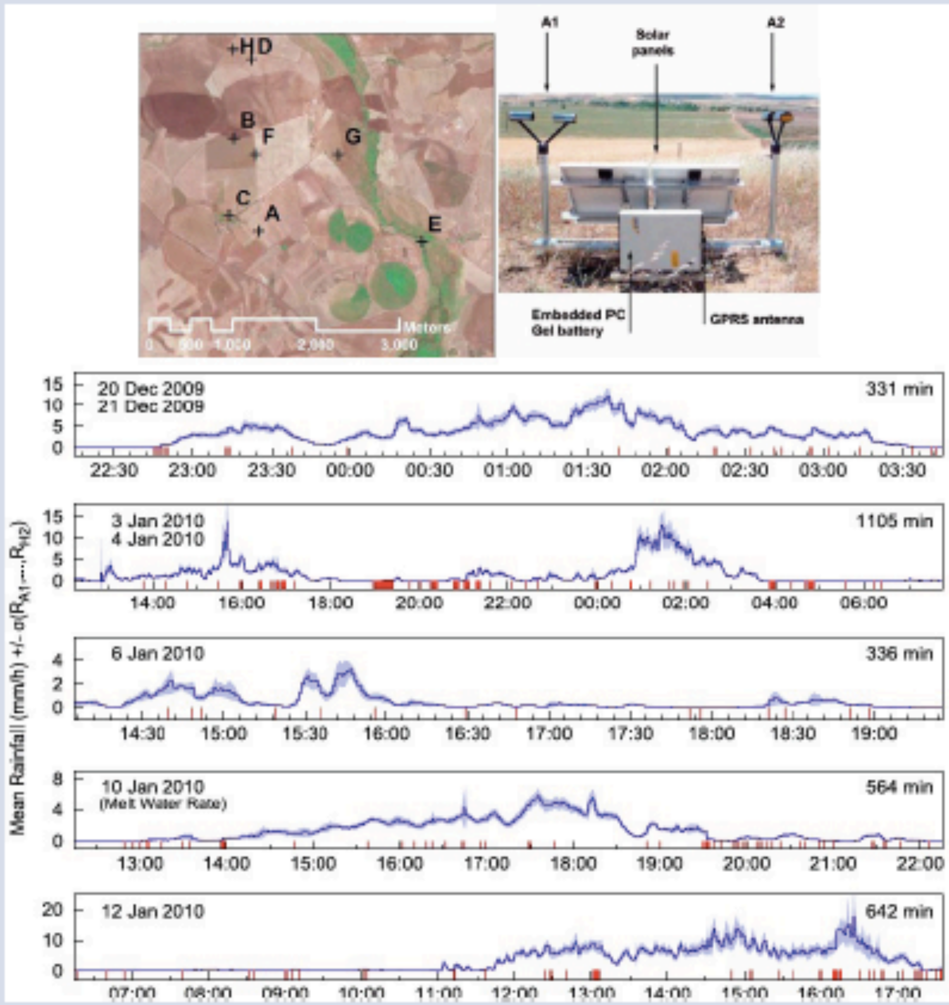
depend on N_T

The proof of the pudding

UCLM database

Experimental setup 1

Medium-scale variability



In 2010, we used 16 Parsivel disdrometers (in a dual setup to ensure consistency) to analyze the spatial variability of the RDSD within a DPR-size pixel.

The experiments were made in central Spain, which has a semiarid climate with moderate rain rates, and thus within Parsivels' known limitations.

As described in the paper below, we found a consistent pattern of DSD variability with distance, and a noticeable spread in the *a* and *b* parameters of the Z/R relationship within the same episode.

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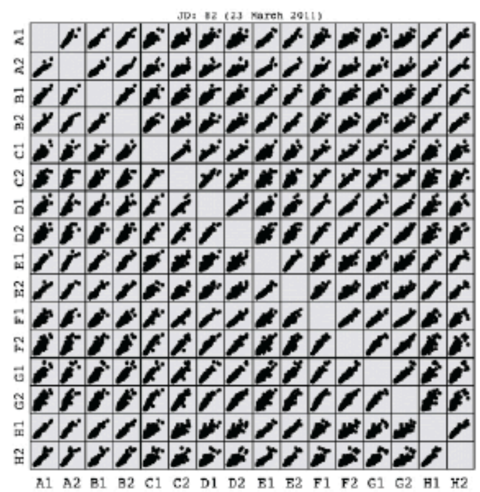
Experimental setup 2

Small-scale variability



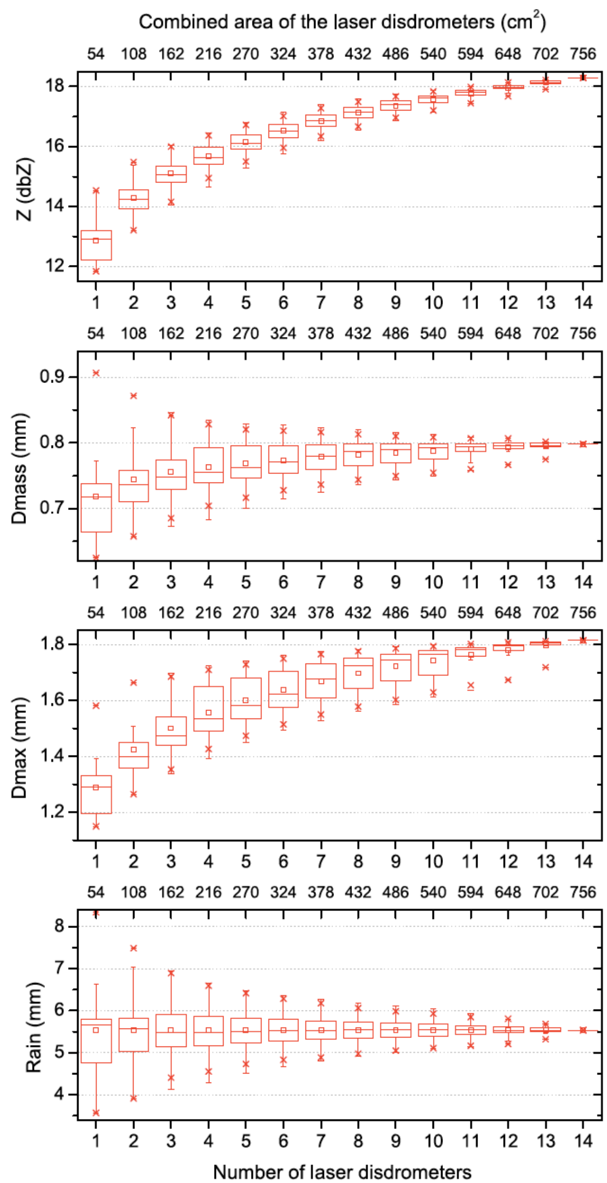
In 2011, we located 16(+2) Parsivels to analyze the consistency of the instruments, the spatial variability of the RDSD at decimeter scale, and to cross-compare the new Parsivel^2 instruments.

The experiments were made in Toledo, and included a sonic anemometer.

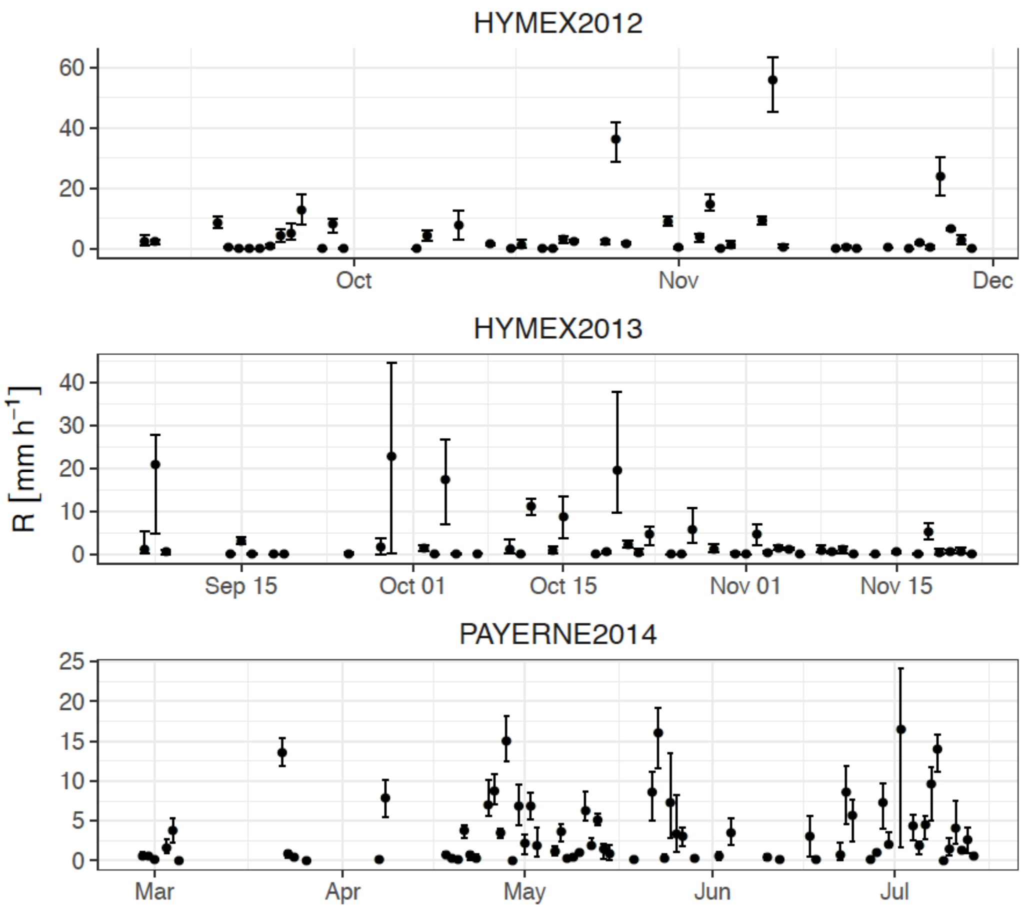
We found that the Parsivels provided consistent estimates of the RDSD for moderate rainfall rates such as those found in Toledo.

Tapiador, F.J., Turk, J., Petersen, W., Hou, A.Y., García-Ortega, E., Machado, L.A.T, Angelis, C.F., Salio, P., Kidd, C., Huffman, G.J. and de Castro, M. 2011. Global Precipitation Measurement: Methods, Datasets and Applications. *Atmospheric Research*, accepted October 2011

Experimental setup 2



EPFL database



Calculations from disdrometer data

	Variable	Equation	Notes
N_T	Total drop concentration [m^{-3}]	$N_T = \int_{Dmin}^{Dmax} n(D)\, dD$	n(D) is the number of drops of diameter D±dD/2
N_0	Gamma PSD intercept [$mm^{-1-\mu} mm^{-3}$]	$n(D) = N_0 D^{\mu} exp(-\Lambda D)$ [N_0 model] $n(D) = N_T \cdot \Lambda^{\mu+1} D^{\mu} \frac{e^{-\Lambda D}}{\Gamma(\mu+1)}$ [Maxent-Gamma model]	For the N_0 model, this corresponds to Gamma parameters fitted to each DSD using the method and (modified) R code of Johnson et al. 2014, which allows for truncated and classed DSDs. For the other models, cfr. Sections 3.1 and 3.2.
Λ	Gamma PSD slope parameter [mm^{-1}]		
μ	Gamma PSD shape parameter [-]		
$E[D]$	First raw moment of the PDF	$E[D] = \int_{Dmin}^{Dmax} D\ \ p(D)dD$	Mean of the PDF
$E[D^2]$	Second raw moment of the PDF	$E[D^2] = \int_{Dmin}^{Dmax} D^2\, p(D)dD$	
$E[\log(D)]$	First raw moment of the PDF	$E[log(D)] = \int_{Dmin}^{Dmax} \log(D)\, p(D)dD$	Logarithmic mean of the PDF
W	Total water content [mm]	$W = \frac{\pi \rho_w 10^{-6}}{6} \int_{Dmin}^{Dmax} D^3\, n(D)dD$	Converted to mm from g m^{-3}, assuming that the water content collects over an area of one m^2. ρ_w[g m^{-3}] is water density.
$F_{N(\mu)}$	Normalised PSD shape factor [-]	$F_{N(\hat{\mu})} = \frac{6}{4^4} \frac{(4+\hat{\mu})^{\hat{\mu}+4}}{\Gamma(\hat{\mu}+4)}$	Calculated using a value of $\hat{\mu}$ fitted to each DSD using the technique of Johnson et al. 2014 for the gamma model, modified such that the slope was always given by $(\hat{\mu}+4)/D_m$.
N_w	Normalised PSD intercept [mm^{-1} mm^{-3}]	$N_w = \frac{4^4}{\pi \rho_w}\left(\frac{10^3 W}{D_m^4}\right)$	
D_m	Mass-weighted mean drop diameter [mm]	$D_m = \frac{\int_{Dmin}^{Dmax} D^4 n(D)\, dD}{\int_{Dmin}^{Dmax} D^3 n(D)\, dD}$	
Z	Radar reflectivity [mm^6 m^{-3}]	$Z = \frac{10^6 \lambda^4}{\pi^5 \lvert K\rvert^2} \int_{Dmin}^{Dmax} \sigma_b(D) n(D)\, dD$	$\sigma_b(D)$[cm^2] is the back-scattering cross-section of a raindrop of equivolume diameter D (in horizontal or vertical polarization). $\lvert K\rvert^2$is the dielectric factor of water. λ is the radar wavelength [cm].
K_{DP}	Specific differential phase shift [deg km^{-1}]	$K_{DP} = \left(\frac{180}{\lambda}\right) 10^{-1} C W (1 - r_m)$	

Three competing models

N_0	MaxEnt Gamma	MaxEnt
$[N_0, \Lambda, \mu]$	$[N_T, E[D], E[log(D)]]$	$[N_T, E[D], E[D^2]]$

Parameter estimation

N_0	MaxEnt Gamma	MaxEnt
$[N_0, \Lambda, \mu]$	$[N_T, E[D], E[log(D)]]$	$[N_T, E[D], E[D^2]]$
Numerical, Maximum Likelihood (MLE)		

Parameter estimation

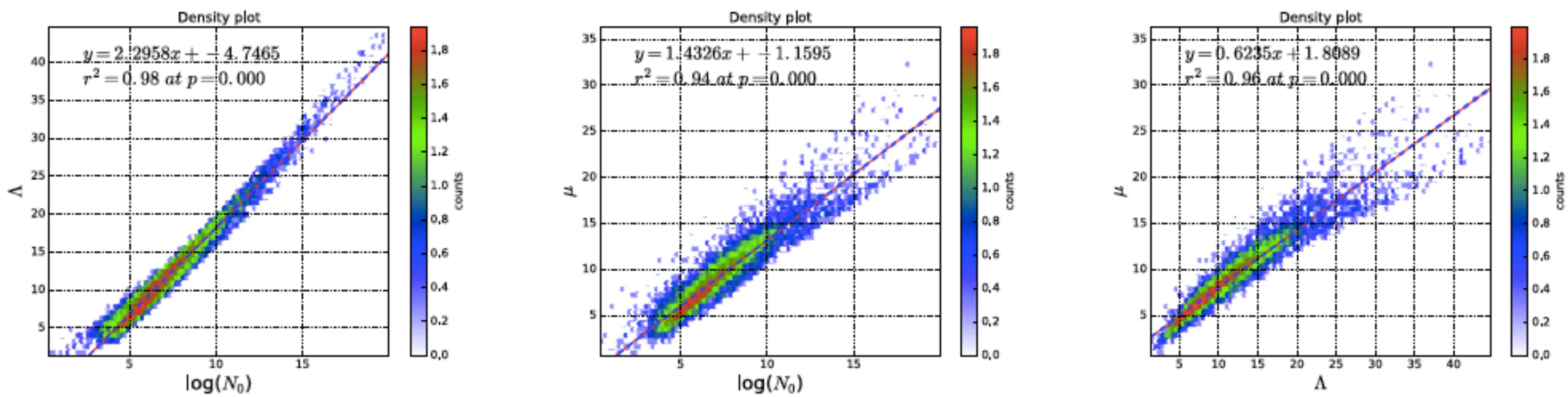
N_0	MaxEnt Gamma	MaxEnt
$[N_0, \Lambda, \mu]$	$[N_T, E[D], E[log(D)]]$	$[N_T, E[D], E[D^2]]$
	$\widehat{\Lambda} = (\hat{\mu} + 1)/\mathrm{E[D]}$	
Numerical, Maximum Likelihood (MLE)	$\hat{\mu} = \frac{1 \pm \sqrt{1+\frac{4}{3}c}}{4c} - 1; c \neq 0$	
	$\mathrm{c} \equiv \log(E[D]) - E[logD]$	

Parameter estimation

N_0	MaxEnt Gamma	MaxEnt
$[N_0, \Lambda, \mu]$	$[N_T, E[D], E[log(D)]]$	$[N_T, E[D], E[D^2]]$
Numerical, Maximum Likelihood (MLE)	$\widehat{\Lambda} = (\hat{\mu} + 1)/\mathrm{E[D]}$ $\hat{\mu} = \frac{1 \pm \sqrt{1+\frac{4}{3}c}}{4c} - 1; c \neq 0$ $\mathrm{c} \equiv \log(E[D]) - E[logD]$	Numerical, Newton-Raphson approach

Parameters independence

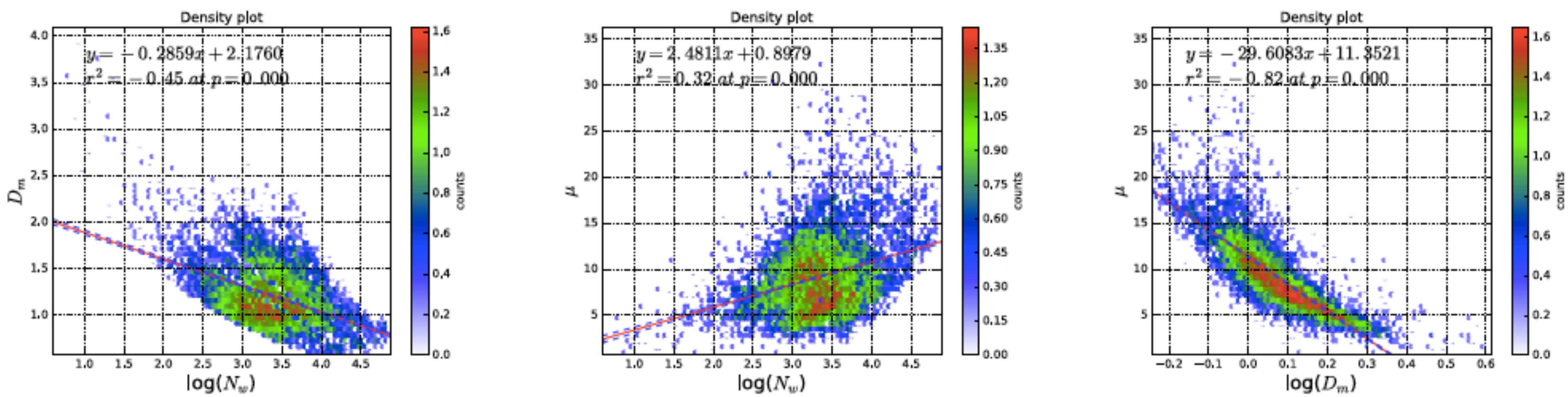
N_0, Λ, μ (N_0 based)



- Number concentration and shape are mathematically related
- Data confirm that indeed are empirically correlated

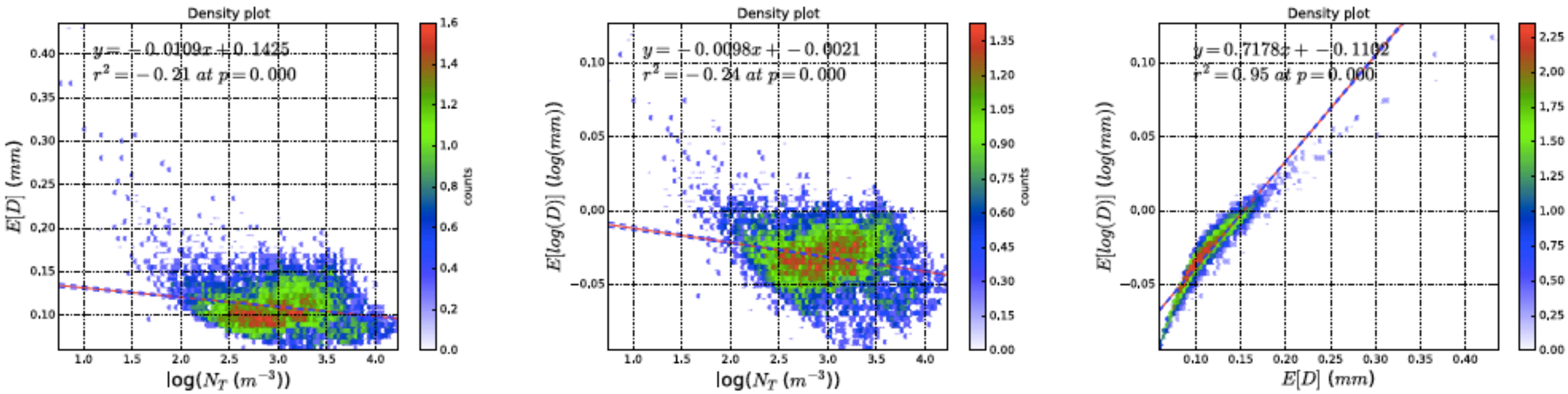
Parameters independence

N_w, D_m, μ (N_w based)

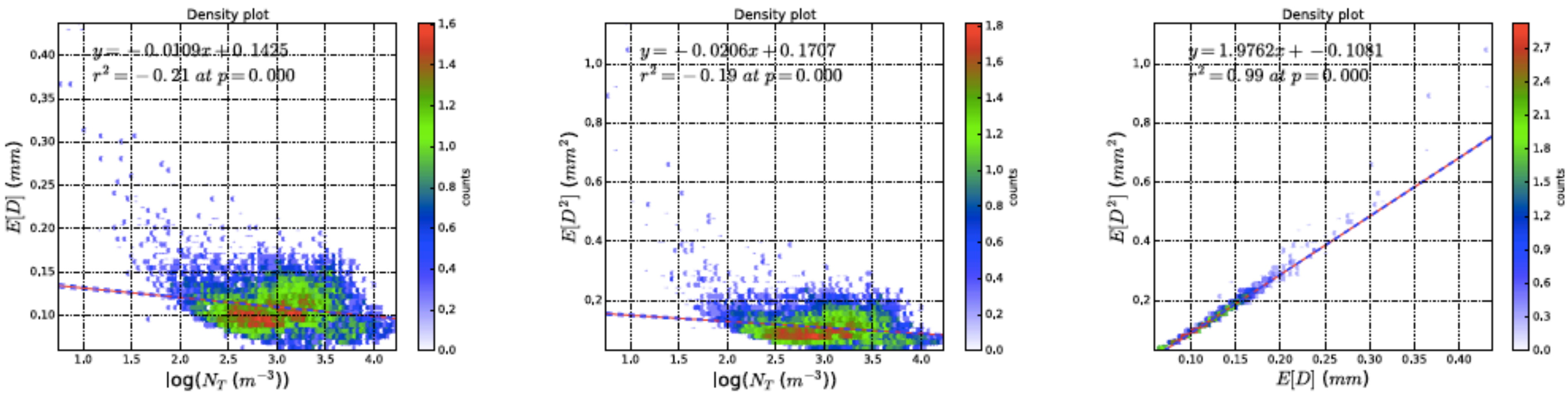


- Number concentration and shape are inherently related
- Data confirms that

N_T, E[D], E[log(D)] (Maxent-Gamma)



N_T, E[D], E[D^2] (Maxent)



But such dependence is sensible for rainfall!

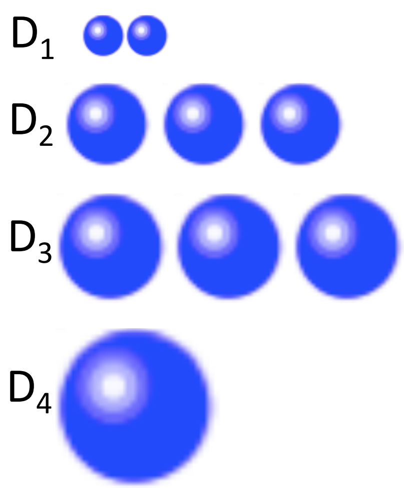
- Number of drops does no depend on drop size probabilities!

- If the shape moves to the right (increase in mean diameter m), then the PDF widens

- We can reduce the number of parameters, but in the shape side only

RDSD #1

N_T=9



RDSD #1

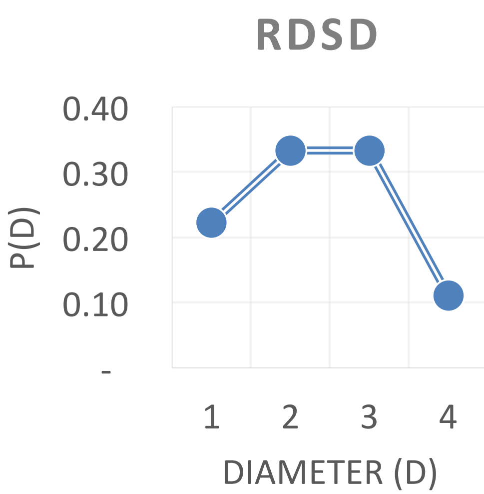
N_T=9

D_1

D_2

D_3

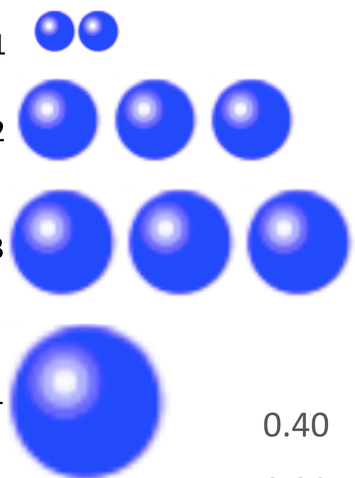
D_4

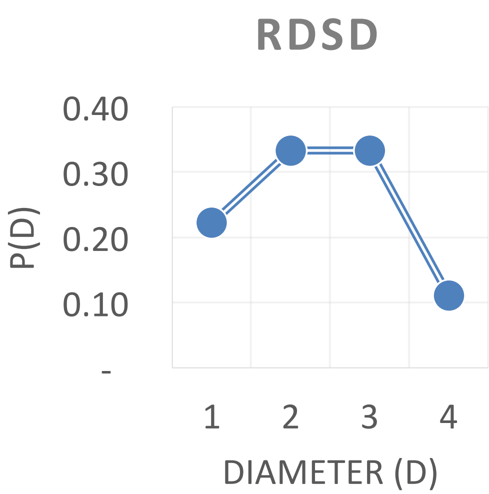


RDSD #1

N_T=9

D_1
D_2
D_3
D_4

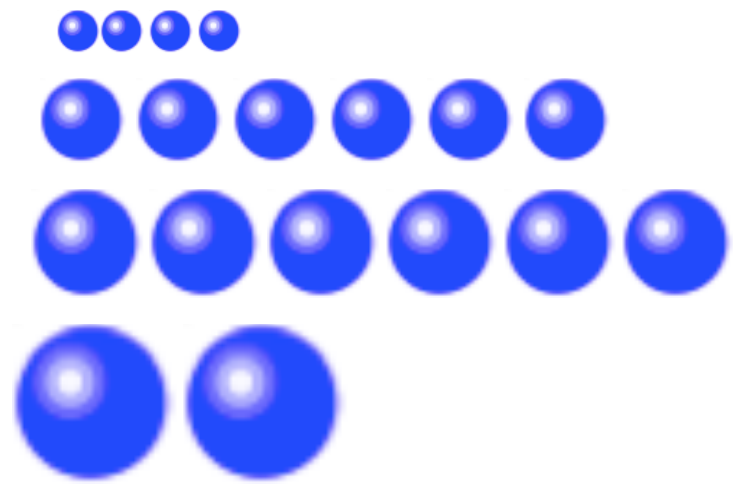




RDSD #2

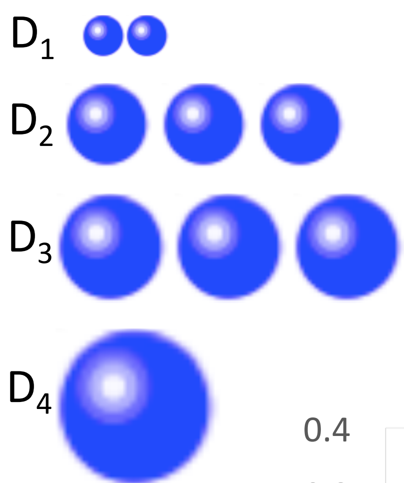
N_T=18

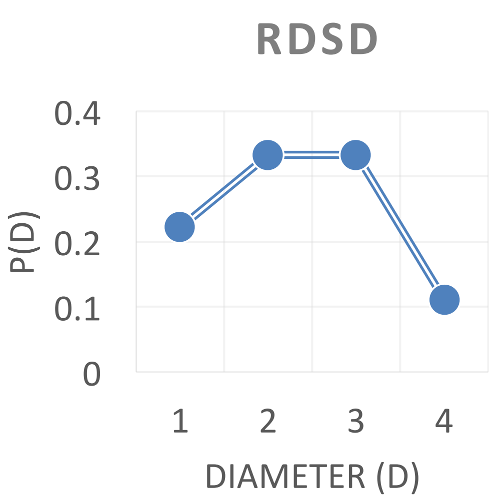
D_1
D_2
D_3
D_4



RDSD #1

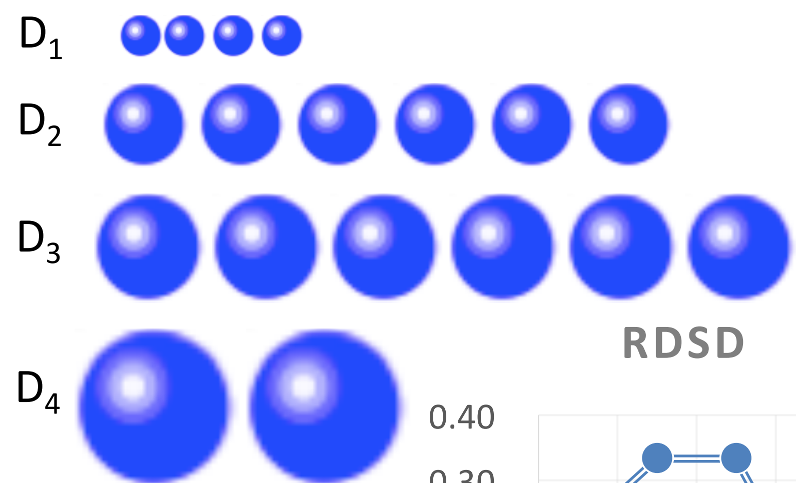
N_T=9

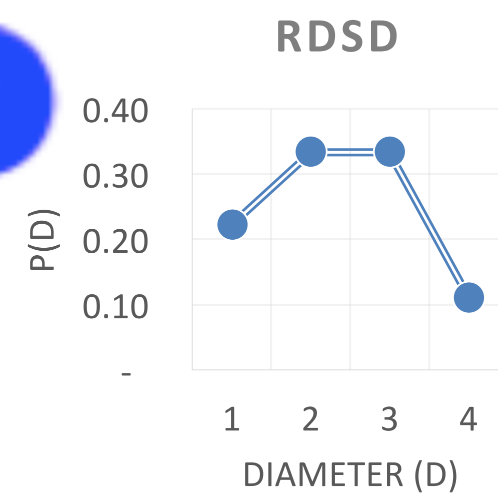




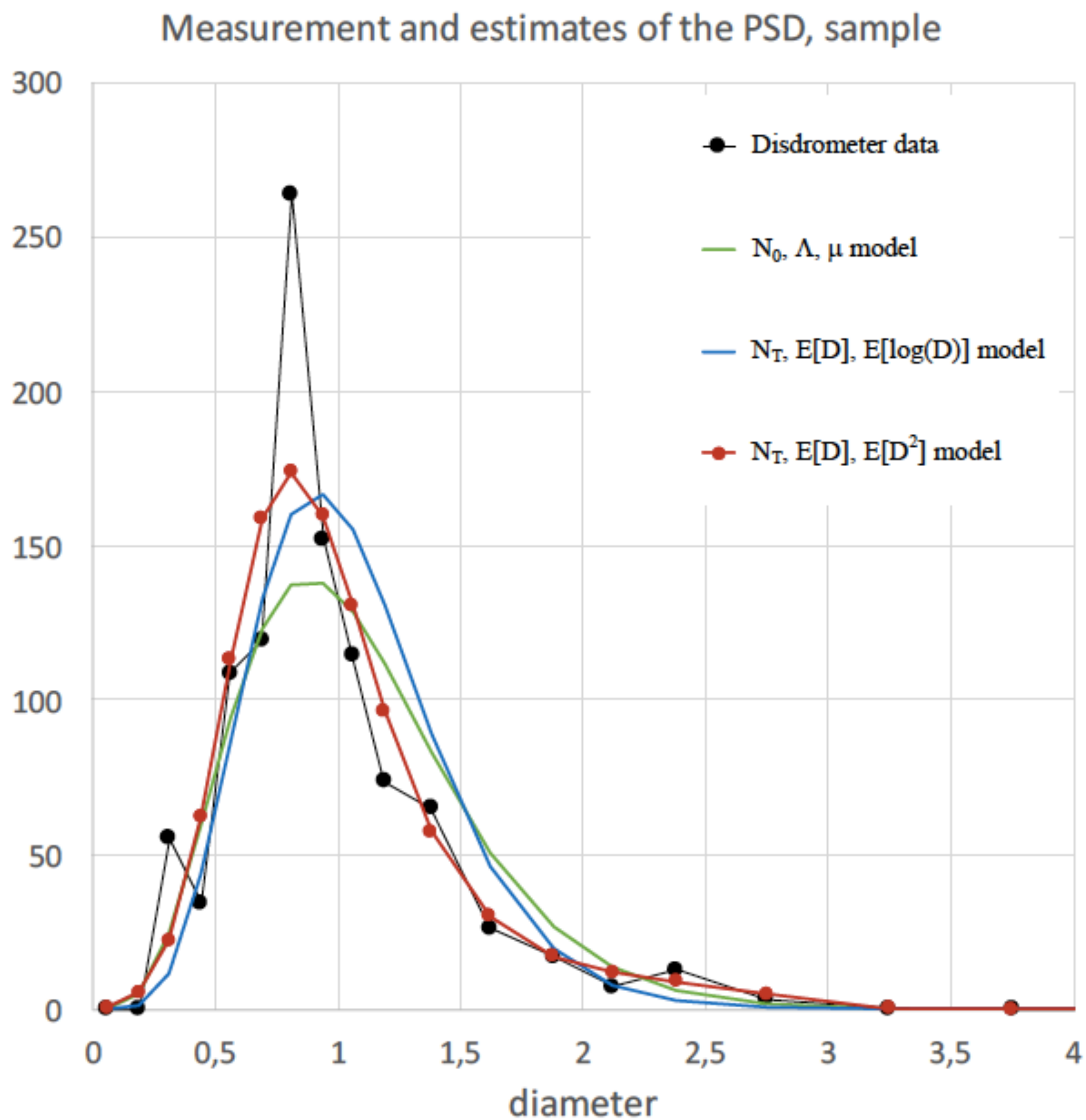
RDSD #2

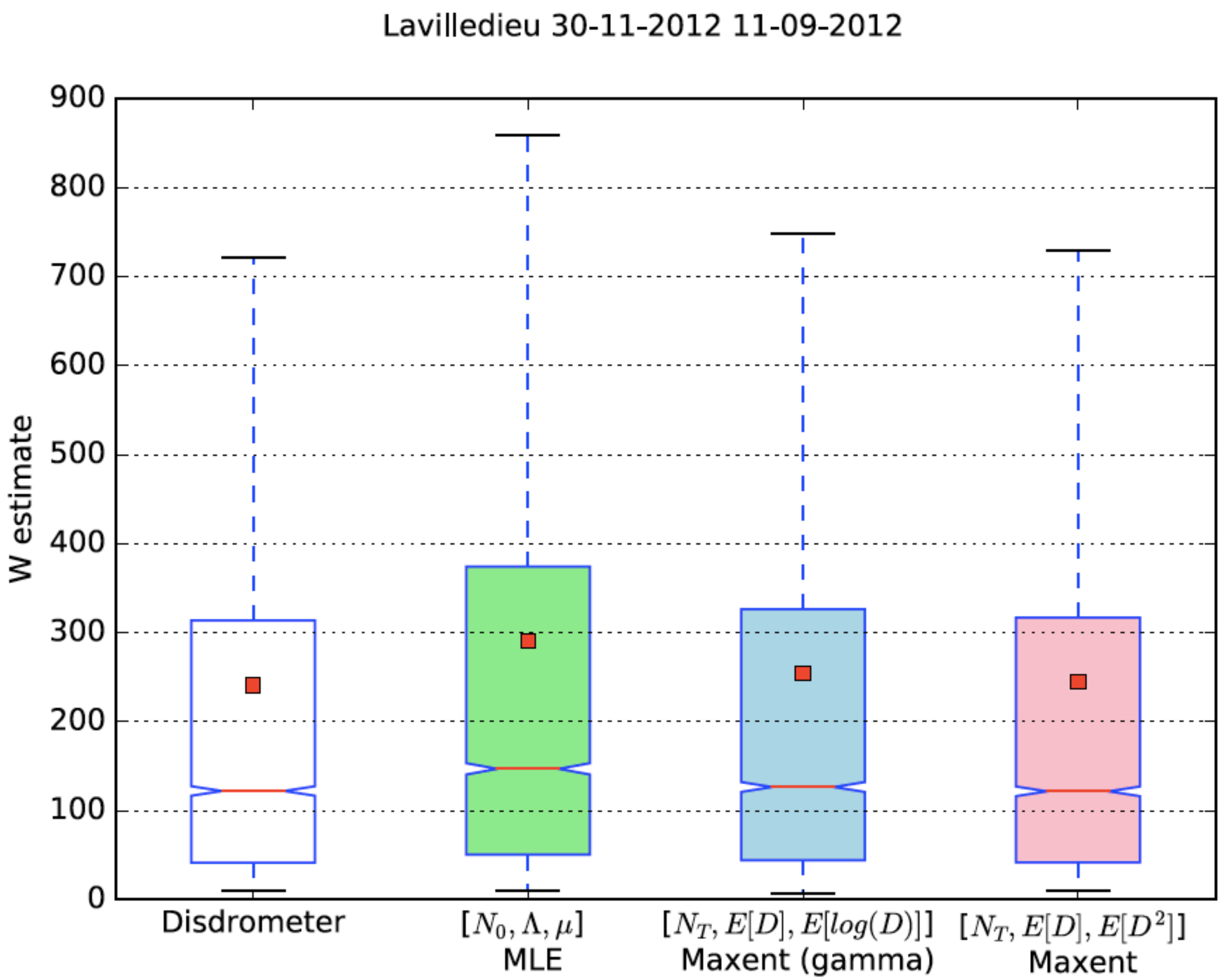
N_T=18

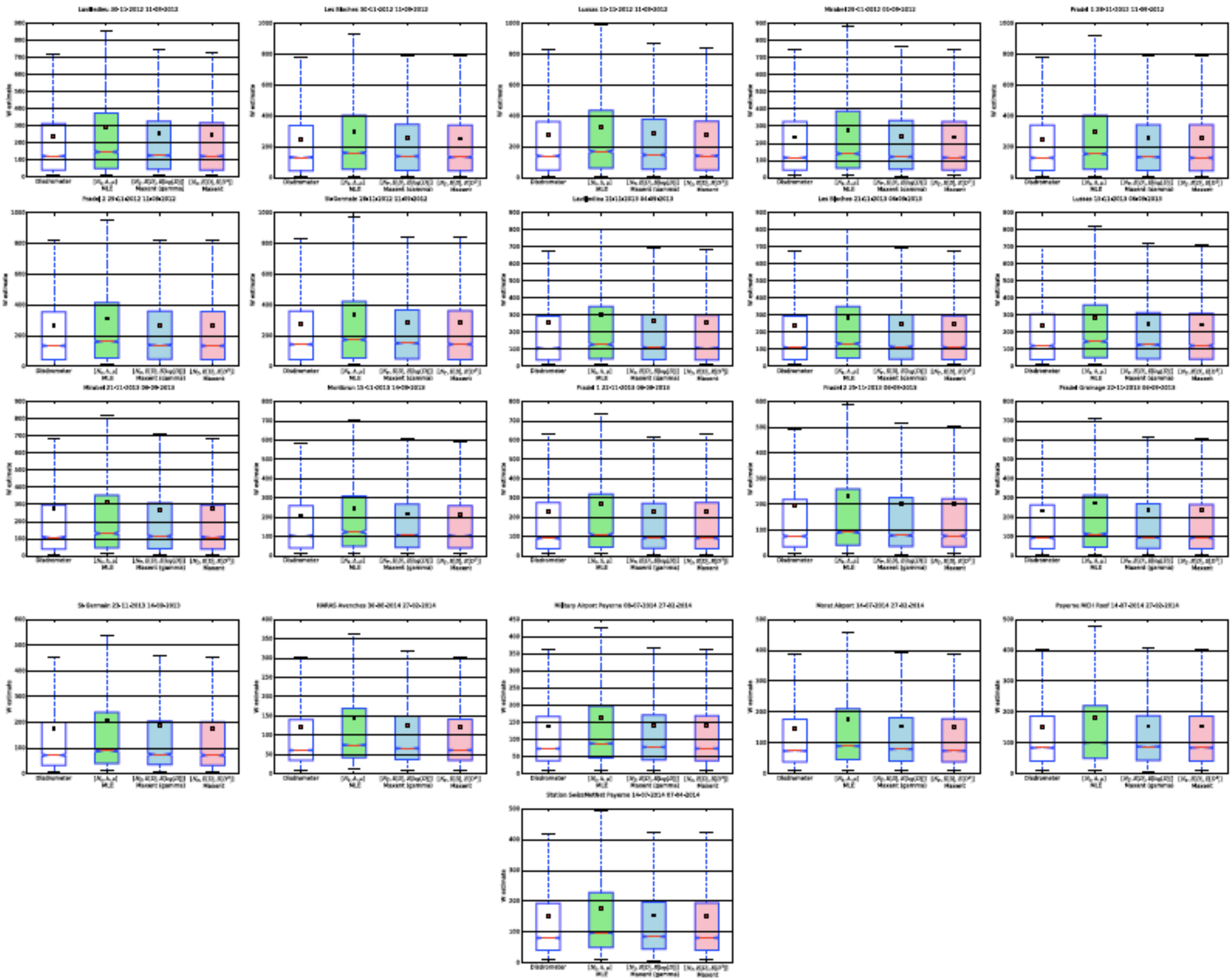


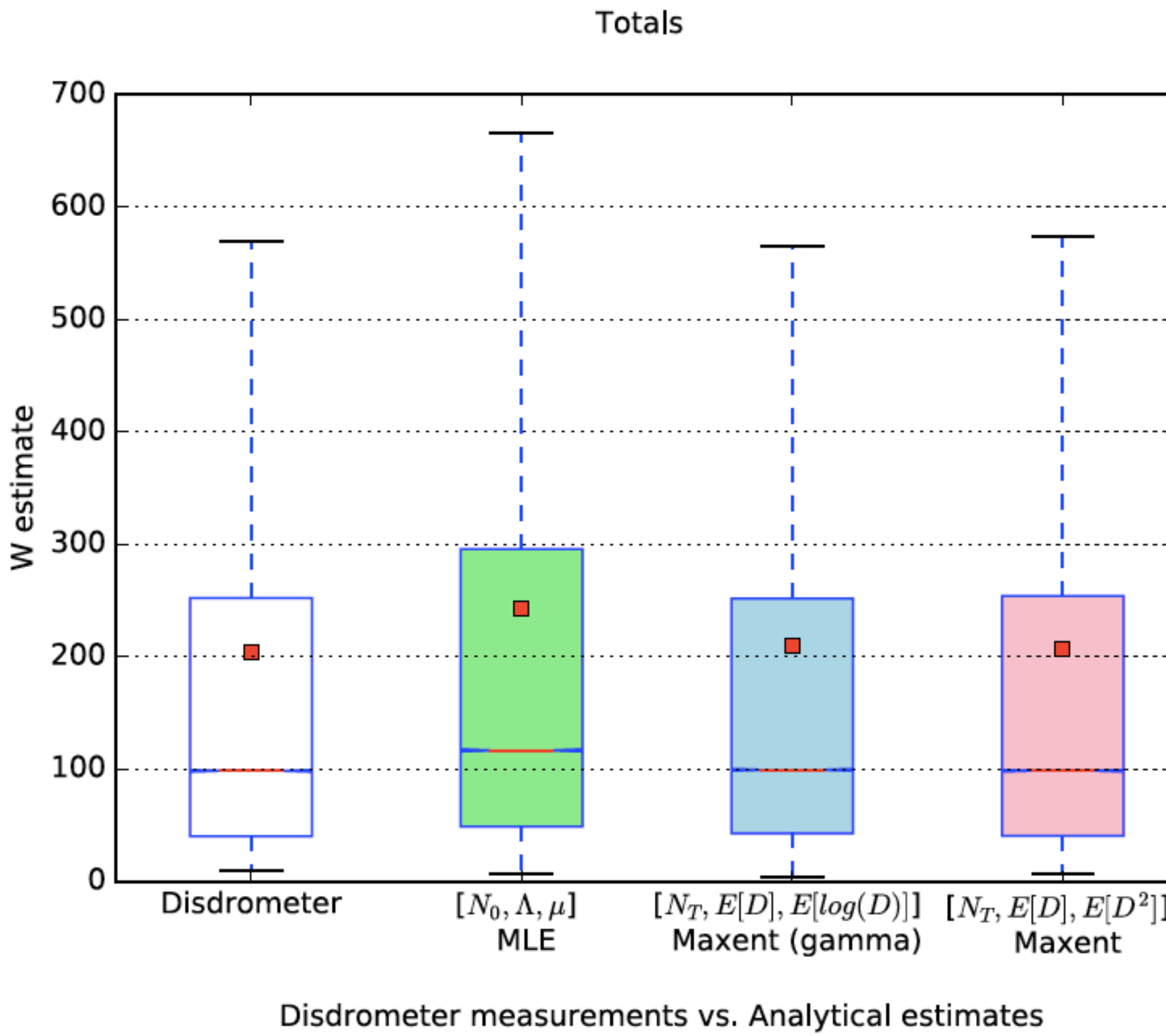


How good are the three models in estimating the actual shape?




Lavilledieu 30-11-2012 11-09-2012
W estimate
0
100
200
300
400
500
600
700
800
900
Disdrometer
$[N_0, \Lambda, \mu]$
MLE
$[N_T, E[D], E[log(D)]]$
Maxent (gamma)
$[N_T, E[D], E[D^2]]$
Maxent



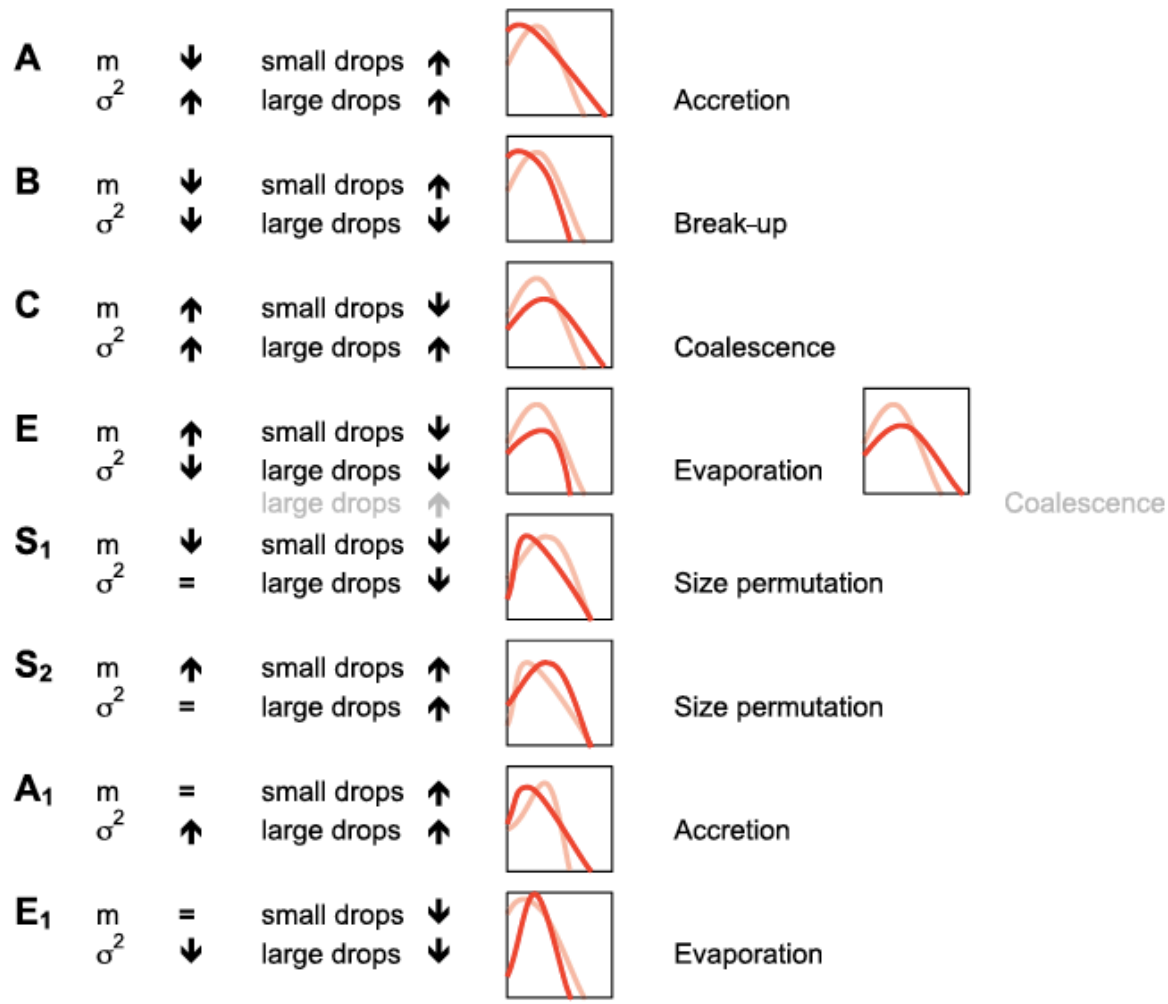

Totals
W estimate
700
600
500
400
300
200
100
0
Disdrometer
$[N_0, \Lambda, \mu]$
MLE
$[N_T, E[D], E[log(D)]]$
Maxent (gamma)
$[N_T, E[D], E[D^2]]$
Maxent
Disdrometer measurements vs. Analytical estimates

Objective mp characterization

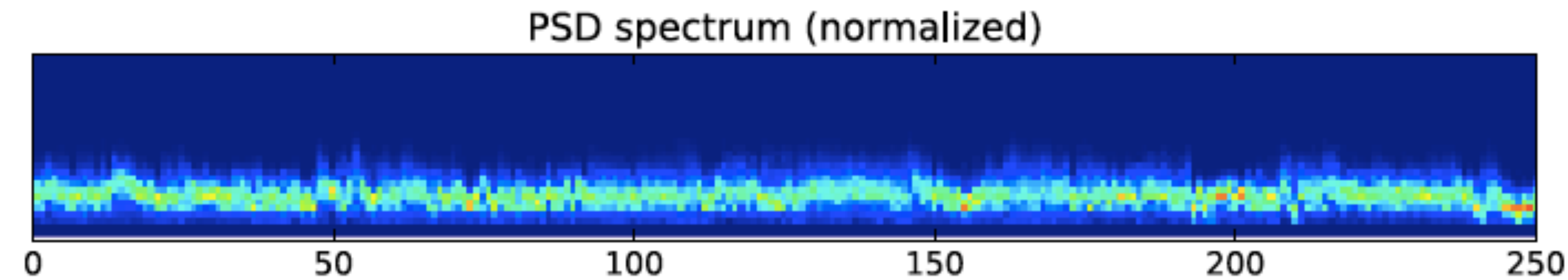
mp processes

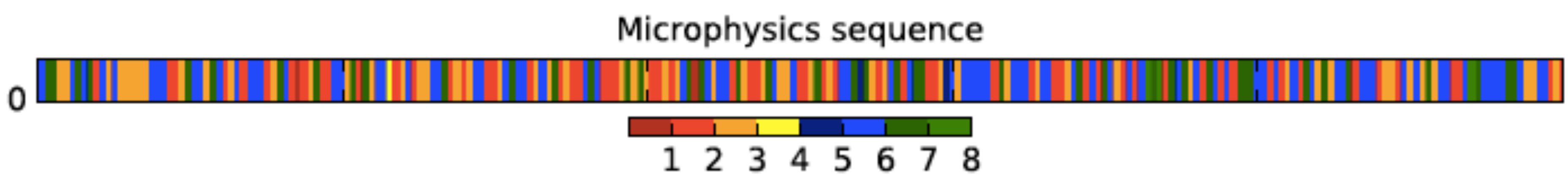
Code	*Coding in Tapiador et al. 2014*	*Microphysics*	*Objective characterization of the microphysical process*	*Comments*
1	A_1	**Accretion**	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} > 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D]}{\partial t} \sim 0$, all drops increase in size
2	B	**Break-up**	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} < 0$	The main process creating new drops
3	S_1	Size sorting 1	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} > 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D^2]}{\partial t} \sim 0$
4	S_2	Size sorting 2	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} < 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D^2]}{\partial t} \sim 0$
5	E_1	**Evaporation 1**	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} < 0$	Process destroying drops
6	C	**Coalescence**	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} > 0$	Major process destroying drops
7	E	Evaporation	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} < 0$	Process destroying (small) drops
8	A	Break-up+ Coalescence	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} > 0$	Large drops break; small drops coalesce

mp characterization



mp characterization





Code	Coding in Tapiador et al. 2014	Microphysics	Objective characterization of the microphysical process	Comments
1	A_1	**Accretion**	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} > 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D]}{\partial t} \sim 0$, all drops increase in size
2	B	**Break-up**	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} < 0$	The main process creating new drops
3	S_1	Size sorting 1	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} > 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D^2]}{\partial t} \sim 0$
4	S_2	Size sorting 2	$\frac{\partial N_T}{\partial t} > 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} < 0$	$\frac{\partial N_T}{\partial t} \sim 0, \frac{\partial E[D^2]}{\partial t} \sim 0$
5	E_1	**Evaporation 1**	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} < 0$	Process destroying drops
6	C	**Coalescence**	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} > 0$	Major process destroying drops
7	E	Evaporation	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} > 0, \frac{\partial E[D^2]}{\partial t} < 0$	Process destroying (small) drops
8	A	Break-up+ Coalescence	$\frac{\partial N_T}{\partial t} < 0, \frac{\partial E[D]}{\partial t} < 0, \frac{\partial E[D^2]}{\partial t} > 0$	Large drops break; small drops coalesce

Retrieval of DSD parameters from radar observables

T-matrix calculations from disdrometer data

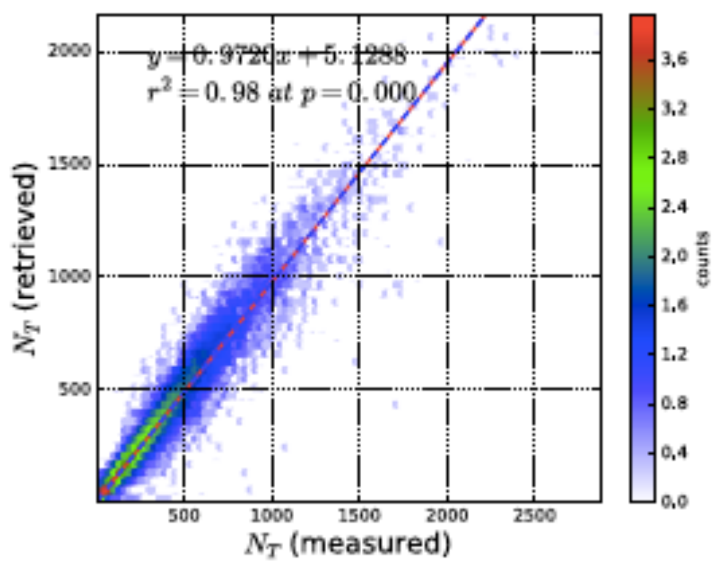
$$\vec{J} = \frac{1}{2}\left(\vec{x} - \vec{Z}\right)^2$$

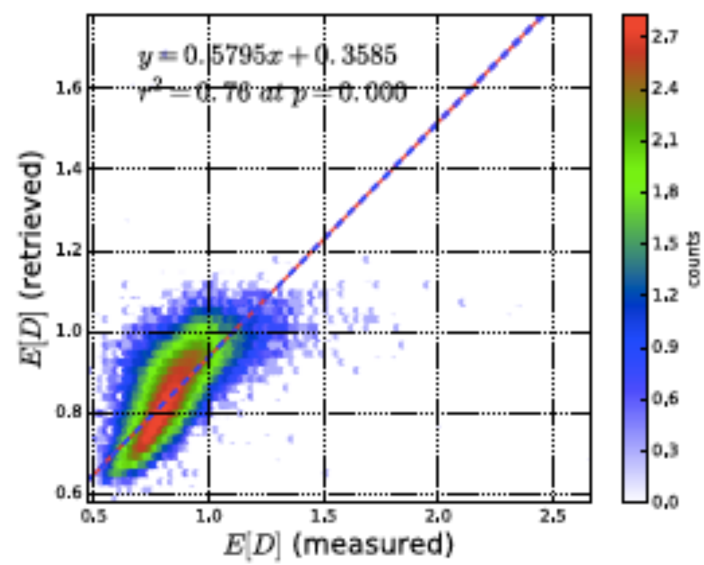
$$\vec{x} = \mathrm{g} * \{\boldsymbol{A} \cdot [\mathrm{g} * (\boldsymbol{B} \cdot \vec{Z})]\}$$

$$\mathrm{g(x)} = 1/(1+\mathrm{e}^{-\mathrm{x}})$$

Retrieval of RDSD params

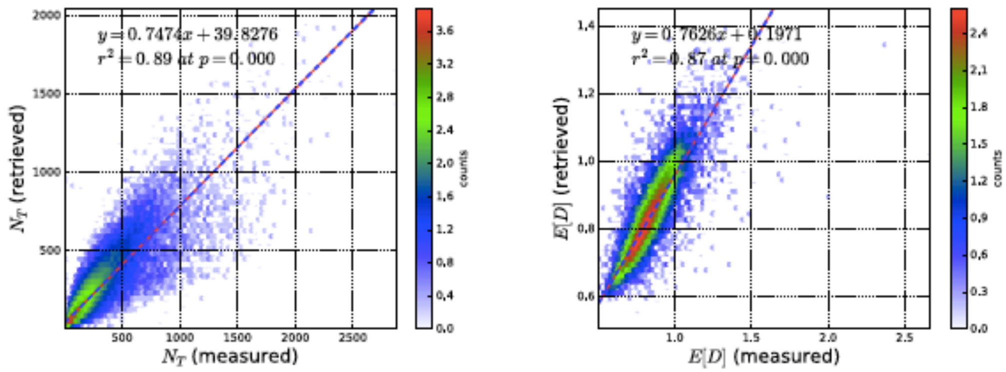
$$\vec{\mathbf{Z}} = (\mathbf{Z_H}(\mathbf{Ka}), \mathbf{Z_V}(\mathbf{Ka}), \mathbf{Z_H}(\mathbf{Ku}), \mathbf{Z_V}(\mathbf{Ku}), \mathbf{Z_{DR}}(\mathbf{Ku}), \mathbf{Z_{DR}}(\mathbf{Ka}), \mathbf{K_{DP}}(\mathbf{Ka}), \mathbf{K_{DP}}(\mathbf{Ku}))$$



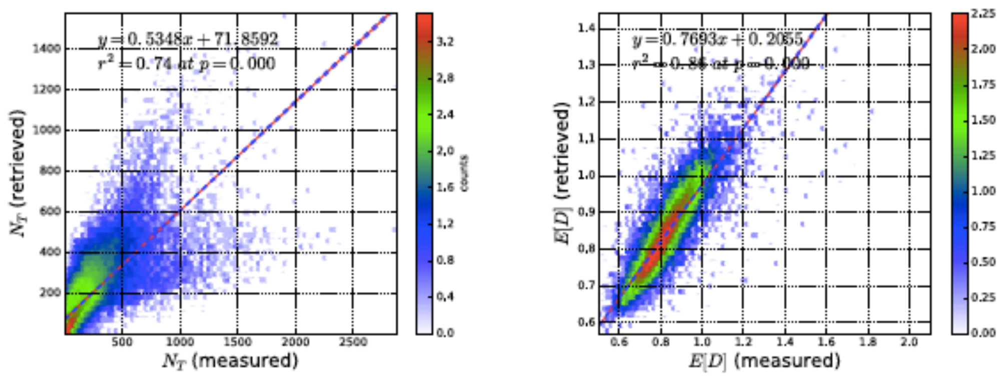


Retrieval of RDSD params

$$\vec{\mathbf{Z}} = \left(\mathbf{Z_V}(\mathbf{Ka}), \mathbf{Z_H}(\mathbf{Ku}), \mathbf{Z_{DR}}(\mathbf{Ku}), \mathbf{Z_{DR}}(\mathbf{Ka})\right)$$



$$\vec{\mathbf{Z}} = \left(\mathbf{Z_H}(\mathbf{Ka}), \mathbf{Z_V}(\mathbf{Ka}), \mathbf{Z_H}(\mathbf{Ku})\right)$$



Summary

1. mp modeling asks for decoupling

   N_T (number of drops) from the
   Shape (probability of given diameter)
2. Both N_0 and N_W models present issues
3. Parameter independence is important in DSD modeling
4. The N_T model is conceptually stronger
5. The N_T model compares better with observations
6. The N_T model is better suited for remote retrieval (independence)