

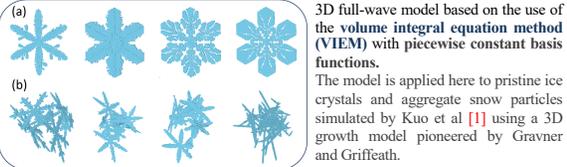
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I – Introduction :

We have developed a 3D full-wave model for Numerically Efficient calculation of Scattering by Complex Particles (NESCoP). The key idea behind NESCoP is the use of a powerful direct solver-based domain decomposition method, known as the Characteristic Basis Function Method (CBFM). NESCoP maintains the advantages of the DDA, namely full-wave solution to arbitrarily-shaped scatterer with inhomogeneous composition, while significantly surpassing the DDA implementations in computational efficiency, particularly when considering a large number of particle orientations. A wide spectrum of enhancements is worth considering to further optimize the numerical efficiency of NESCoP and to improve its accuracy.

II – NESCoP : Efficient CBFM-based Scattering Model



3D full-wave model based on the use of the volume integral equation method (VIEM) with piecewise constant basis functions. The model is applied here to pristine ice crystals and aggregate snow particles simulated by Kuo et al [1] using a 3D growth model pioneered by Gravner and Griffith.

Frequencies of interest : (14 - 800 GHz)

Integral representation of the total electric field (EFIE) :

$$E(r) = E^i(r) + (k_0^2 + \nabla \cdot \nabla) \int_{\Omega} \chi(r') G(r, r') E(r') dr'$$

where $E(r)$ is the field inside the scatterer, $E^i(r)$ is the incident field, $\chi(r')$ is the dielectric contrast at the location r' , k_0 is the wavelength number in air and $G(r, r')$ is the free space Green's function. We rewrite the integral equation above as :

$$[\Gamma E(r) = E^i(r)] \text{ where } \Gamma = \mathbf{I} - (k_0^2 + \nabla \cdot \nabla) \int_{\Omega} \chi(r') G(r, r') dr'$$

Application of a Method of Moments (MoM) :

The particle is discretized into N cubic cells Ω_n of side c_n , $c_n = \frac{\lambda_s}{10}$; $\lambda_s = \frac{\lambda_0}{\sqrt{\text{Re}(\epsilon_r)}}$ small enough to consider that the field inside is constant [2]

$$E(r) = \sum_{n=1}^N \sum_{q=1}^3 E_n^q F_n^q(r) \text{ where } E_n^q \text{ is the constant unknown of } F_n^q, \text{ the } n^{\text{th}} \text{ basis function for the component (q=x, y or z) of the field inside the particle.}$$

To select a set of test functions W_p^m ($m=1, \dots, M$ and $p=x, y, z$), the point matching method is used. So, $M=N$ and W_p^m is a Dirac delta function concentrated at the center of the cell Ω_n .

$$[\Gamma E(r) = E^i(r)] \rightarrow \sum_{n=1}^N \sum_{q=1}^3 \langle W_p^m, \Gamma F_n^q \rangle E_n^q = \langle W_p^m, E^i \rangle$$

$$\text{where } F_n^q(r) = \begin{cases} 1; & r \in \Omega_n \\ 0; & r \notin \Omega_n \end{cases} \hat{q}$$

$$\text{and } \overline{W_p^m}(r) = \delta(r - \overline{r}_m) \hat{p}$$

$$\sum_{n=1}^N \sum_{q=1}^3 Z_{pq}^{mn} E_n^q = E_p^i \rightarrow ZE = E^i$$

The elements of $Z = [Z_{pq}^{mn}]$ are given by $Z_{pq}^{mn} = \delta_{mn} \delta_{pq} - Z_{pq}^{s, mn}$ where

$$Z_{pq}^{s, mn} = k_0^2 \int_{\Omega_n} \chi(\overline{r}'_n) G_{pq}^s(\overline{r}_m, \overline{r}'_n) dr'_n + \sum_{q=1}^3 \frac{\partial^2}{\partial p_m \partial q_n} \int_{\Omega_n} \chi(\overline{r}'_n) G_{pq}^s(\overline{r}_m, \overline{r}'_n) dr'_n$$

If $m=n$, in order to avoid singularities, $Z_{pq}^{s, mn}$ is computed using Hadamard regularization. Then the integral on Ω_n is approximated by an integral on sphere of radius $a_n = c_n \sqrt{3}/(4\pi)$.

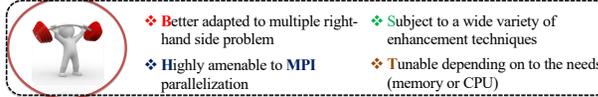
$$\text{if } m = n : Z_{pq}^{s, mn} = \mathcal{H} \left(\int_{\Omega_n} \left(k_0^2 + \frac{\partial^2}{\partial p_m^2} \right) G_{pq}^s(\overline{r}_m, \overline{r}'_n) dr'_n \right) \chi_n \text{ if } p=q; 0 \text{ if } p \neq q$$

$$= \frac{2}{3} e^{jk_0 a_n} (1 - jk_0 a_n) - 1$$

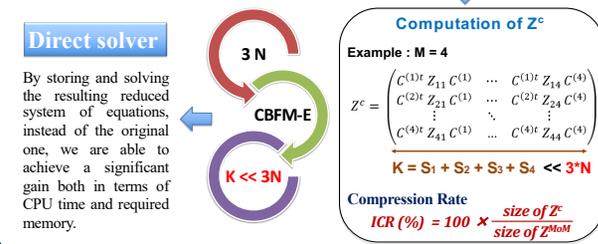
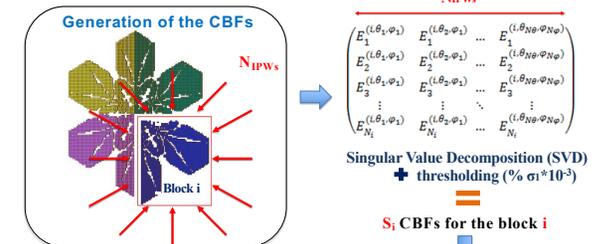
$$\text{if } m \neq n : \int_{\Omega_n} G_{pq}^s(\overline{r}_m, \overline{r}'_n) dr'_n = 4\pi \times \frac{\sin(k_0 a_n) - k_0 a_n \cos(k_0 a_n)}{k_0^3} \times G_{pq}^s(\overline{r}_m, \overline{r}'_n)$$

Application of the Characteristic Basis Function Method :

To overcome the computational burden associated with the VIEM ($O((3N)^3)$), we use the CBFM, a domain-decomposition method proven to be accurate and efficient when applied to large-scale EM problems.



After dividing the 3D complex geometry of the particle of N cells into M blocks, the CBFM process [3] consists in generating S_i Characteristic Basis Functions (CBFs) for each block i in order to generate a final reduced matrix of size $K \times K$ where $K = \sum_{i=1}^M S_i$. This results in a substantial size-reduction of the MoM matrix and enables us to use of a direct method for its inversion.



III – Enhancing the Computational Capabilities of NESCoP :

DDScat	NESCoP			
1 to 4 hours	180 to 59 days	190 id 5 hours	703 id 12 hours	1891 id 23 hours

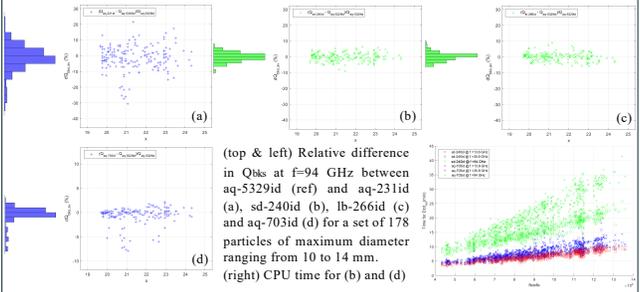
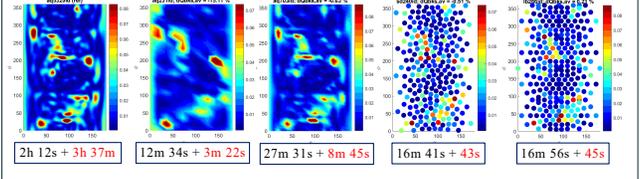
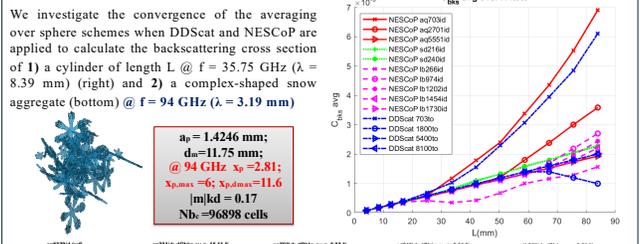
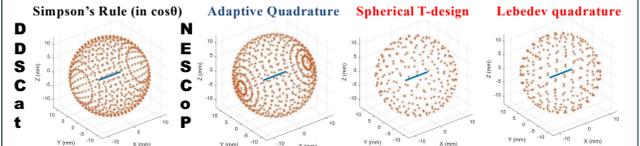
CPU time for NESCoP and DDScat as function of the number of target orientations (to) for DDScat and incident directions (id) for NESCoP

$a_p = 1.61 \text{ mm}; d_m = 11.45 \text{ mm};$
 $\lambda = 6 \text{ mm}; x_p = 1.68;$
 $x_{p,max} = 6; |m|_{kd} = 0.0935$
 $N_b = 140896 \text{ cells}$

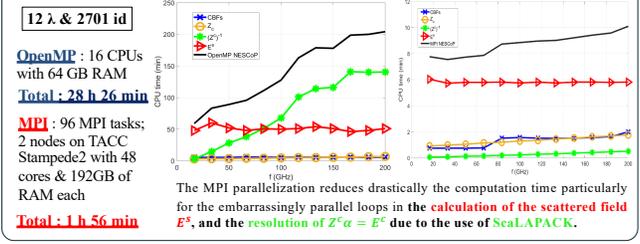
To understand the large relative differences that we are seeing in averaged backscattering quantities between NESCoP and DDScat, we look at the backscattering per target orientation (to) or incident wave direction (id) (θ_i, ϕ_i). It can be clearly seen that the error is not coming from Qbs per to/id.

1. Accurate & Efficient Orientation-Averaging :

We investigate several quadrature schemes for integration over a spherical surface such as Lebedev quadratures and spherical designs and test their performance on different particle shapes and sizes.



2. MPI Parallelization of NESCoP :



References : [1] Kuo, K. S., Olson, W. S., Johnson, B. T., Grecu, M., Tian, L., Clune, T. L., ... & Meneghini, R. (2016). The Microwave Radiative Properties of Falling Snow Derived from Nonspherical Ice Particle Models. Part I: An Extensive Database of Simulated Pristine Crystals and Aggregate Particles, and Their Scattering Properties. Journal of Applied Meteorology and Climatology, 55(3), 691-708. [2] Nguyen, H., Roussel, H., & Tabbara, W. (2006). A coherent model of forest scattering and SAR imaging in the VHF and UHF-band. IEEE transactions on geoscience and remote sensing, 44(4), 838-848. [3] Lucente, G., Tiberti, A., Monorchio, and R. Mitra, "An iteration-free MoM Approach Based on Excitation Independent Characteristic Basis Functions for Solving Large Multiscale Electromagnetic Scattering Problems", IEEE Trans. Antennas Propag., Vol. 56, no. 4, pp.999-1007, Apr. 2008.